Bose-Einstein Condensation : production, fundaments and modern aspects

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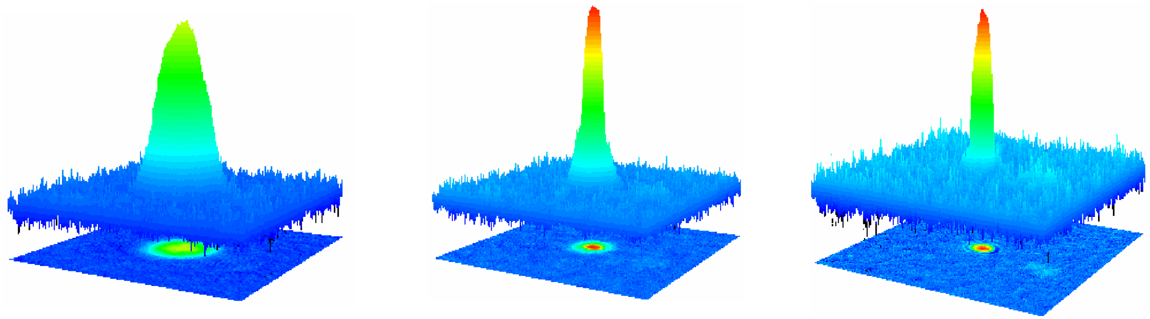


$T > T_c$ $T < T_c$ $T << T_c$

Lecture IV

Lectures:

1) The importance of BEC. Basic concepts for BEC

2) Interactions in BEC/lattices (exercises)

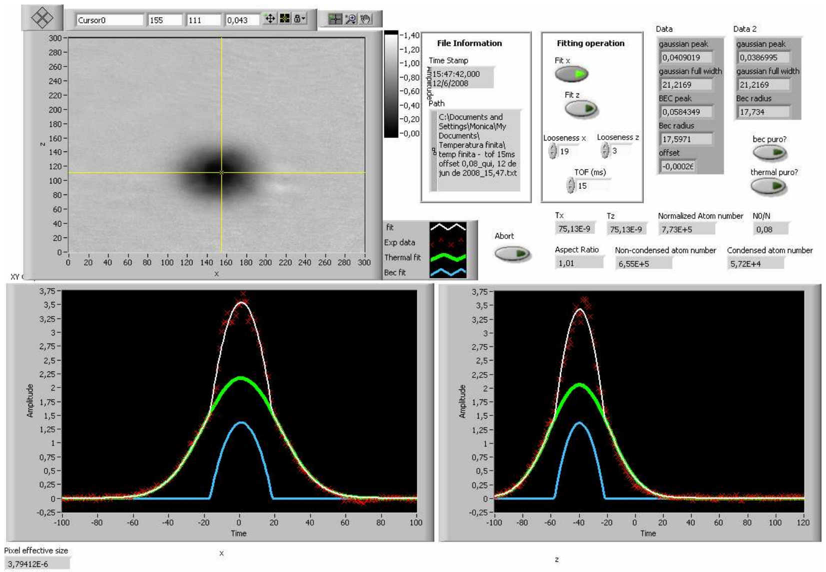
3) How to make BEC? Thermodynamics

4) Coherent modes : equivalence with quantum optics

5) Superfluidity aspects: vortices, turbulence and more

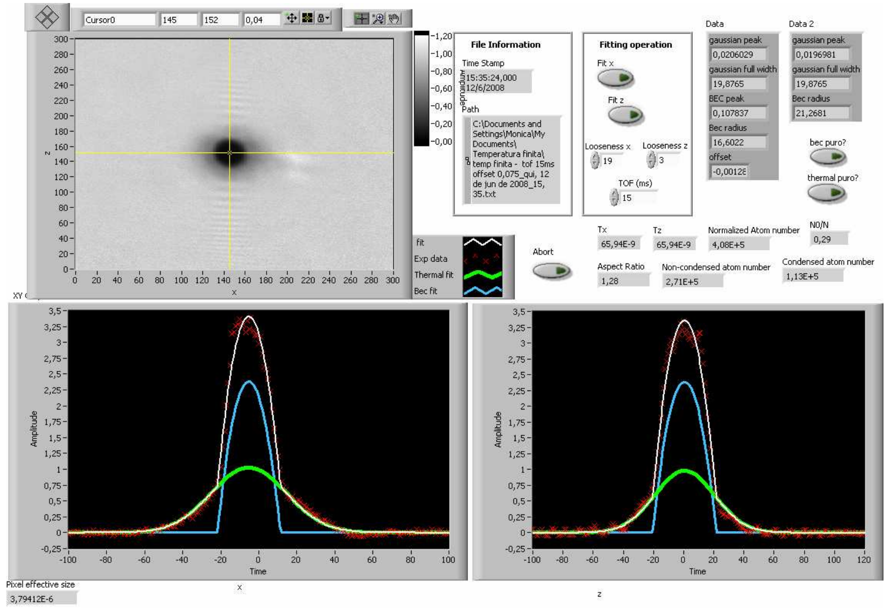
Finite Temperature correction to the Thomas-Fermi approximation for a Bose-Einstein condensate

Image Analysis



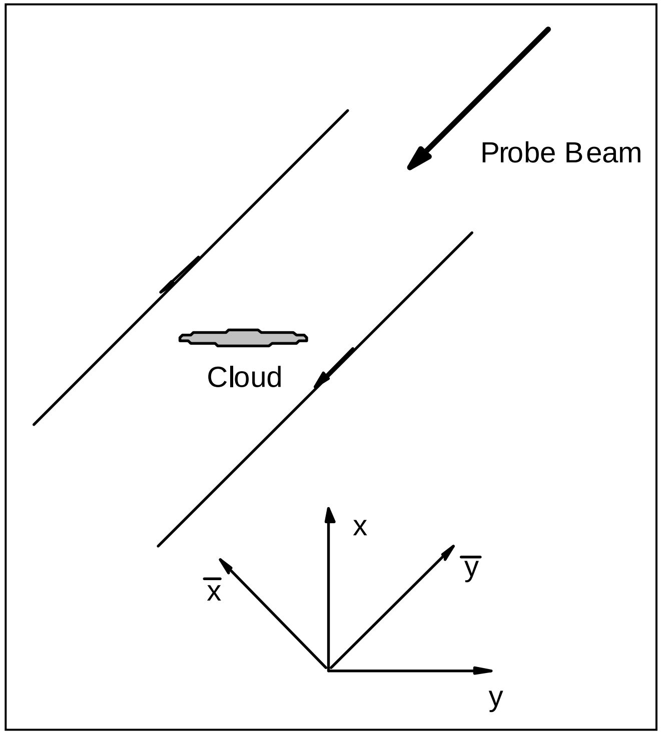
N_o / N = 0,1

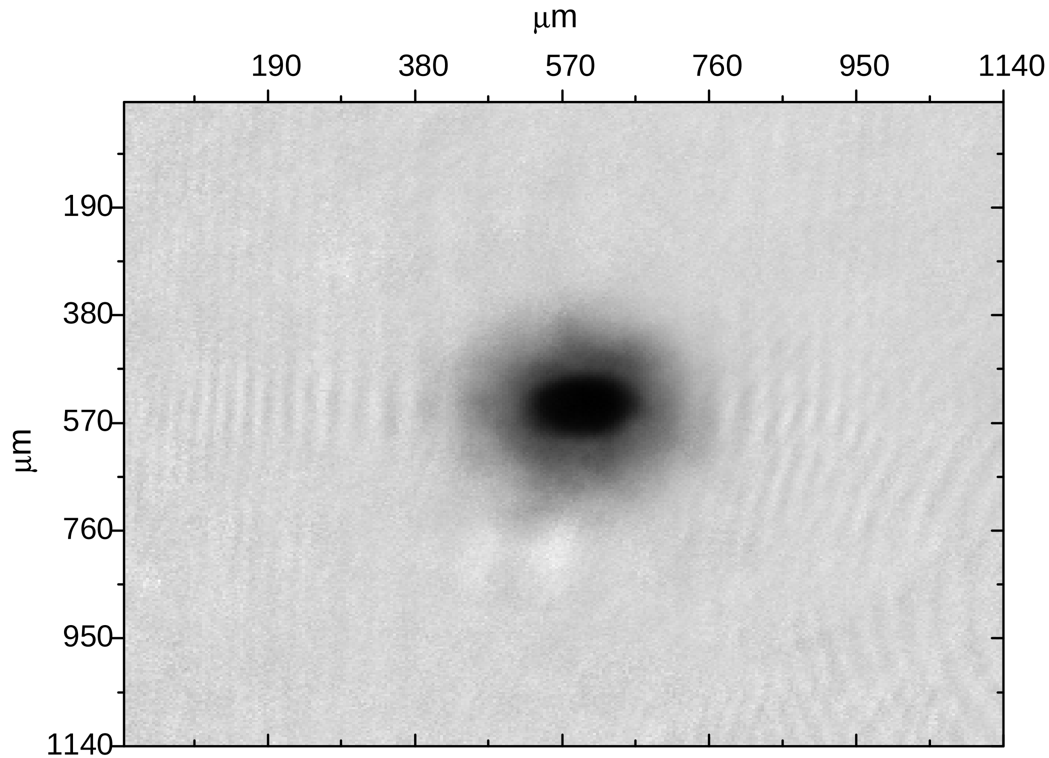
A LabVIEW program allow analysis of the absorption image, separating the condensate cloud from the thermal one.



N_o / N = 0,3

Image Analysis





$$\eta(x,y,z) = \tilde{n}_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2+z^2}{R_z^2}, 0\right) + \tilde{n}_T \, e^{-\frac{x^2+y^2+z^2}{2\sigma^2}}$$

Thomas-Fermi model and T = 0 K

Gross-Pitaevskii Equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) + U_0|\psi(\vec{r})|^2\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$

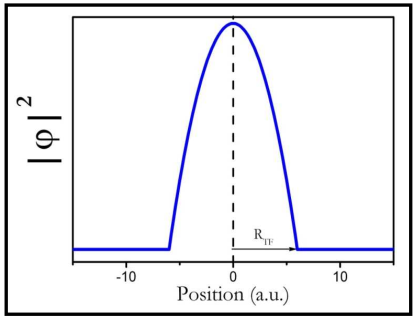
Thomas – Fermi Aproximation

$$\left[V(\vec{r}) + U_0|\psi(\vec{r})|^2\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$

interactions dominant

$$\Rightarrow \quad n_0(\vec{r}) = \frac{\mu - V(\vec{r})}{U_0}$$

$$U_0 = \frac{4\pi\hbar^2 a}{m}$$



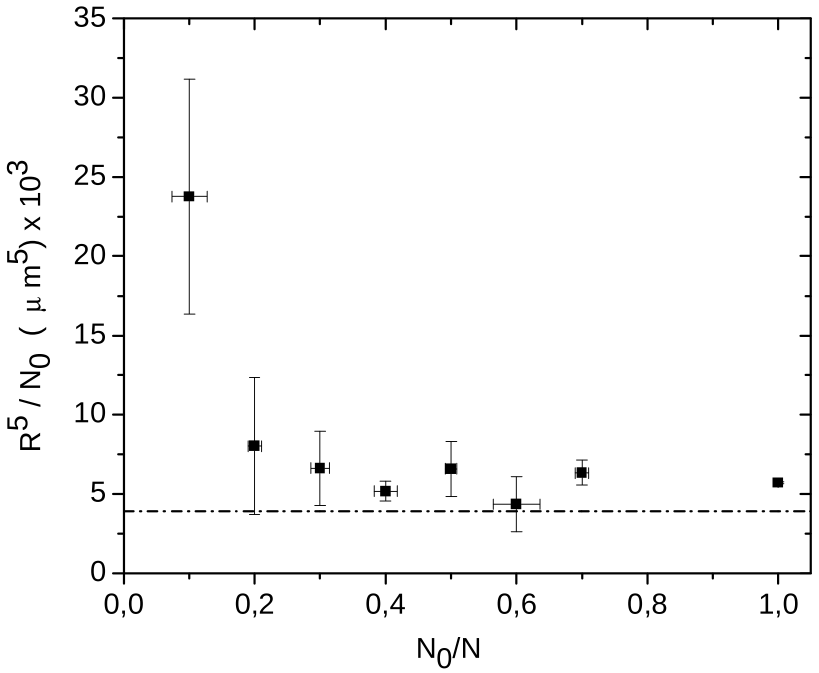
$$n_0(R) = 0$$

$$\mu - V(R) = 0$$

$$\int |\psi(\vec{r})|^2 dr^3 = N_0$$

$$R = \left(\frac{15}{4\pi}\frac{U_0}{m\omega^2}N_0\right)^{1/5} \Rightarrow \boxed{\frac{R^5}{N_0} = C}$$

Results



Different dependence when T ≠ 0
Finite temperature corrections
are necessary

$$\frac{R^5}{N_0} = \frac{15}{4\pi}\frac{U_0}{m\omega^2} = C$$

$$C_{TOF} = \left(\left[1+\varepsilon^2(\tau\arctan\tau - \ln\sqrt{1+\tau^2})\right](1+\tau^2)\right)^{5/3} C$$

$$\tau = \omega_r t \qquad \varepsilon = \omega_z / \omega_r$$

Simple model

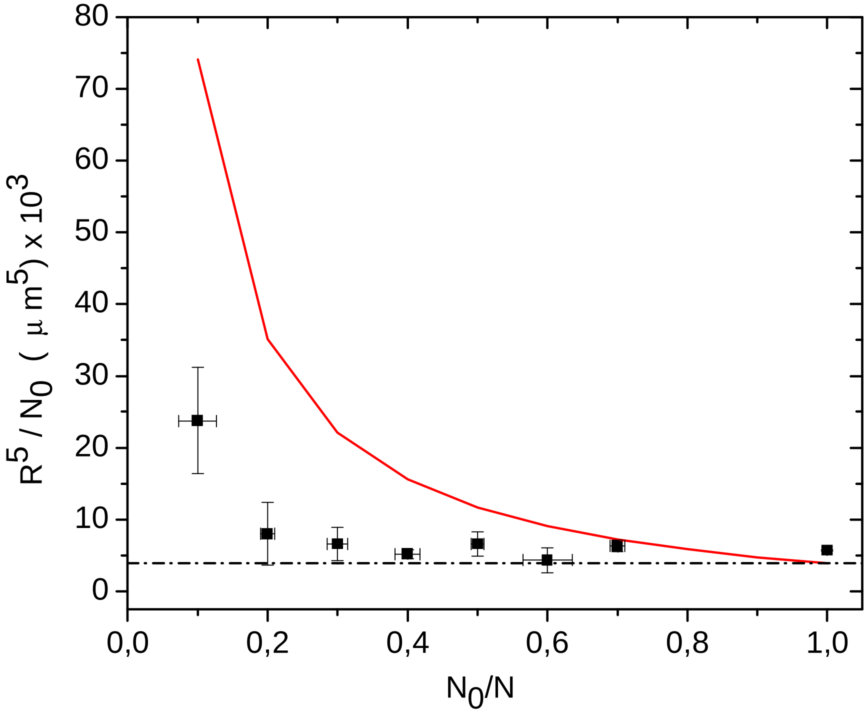
Add a term to proportional to the density of thermal cloud: $2U_0 n_T(\vec{r})$

Thomas – Fermi Aproximation: $n_0(\vec{r}) + 2U_0 n_T(\vec{r}) = \dfrac{\mu - V(\vec{r})}{U_0}$

Integrating over the available space: $N_0 + 2N_T = \dfrac{R^5}{C}$

Using : $N = N_0 + N_T$

$$\Rightarrow \boxed{\frac{R^5}{N_0} = C\left(\frac{2}{\frac{N_0}{N}} - 1\right)}$$



Hartree-Fock model

Semi-classical approximation:

$$n_0(\vec{r}) = \frac{\mu - V(\vec{r}) - 2U_0 n_T(\vec{r})}{U_0} H(\mu - V(\vec{r}) - 2U_0 n_T(\vec{r}))$$

$$n_T(\vec{r}) = \frac{1}{\lambda_T^3} g_{3/2}\left(e^{-\frac{V(\vec{r})+2U_0 n_0(\vec{r})+2U_0 n_T(\vec{r})-\mu}{k_B T}} \right) \qquad g_{3/2}(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^{3/2}}$$

Thermal cloud subjects to an effective potential:

$$V_{eff}(\vec{r}) = V(\vec{r}) + 2U_0 n_T(\vec{r}) + 2U_0 n_0(\vec{r})$$

➡ density distribution for the condensate and thermal cloud obtained through a self-consistent system of two equations

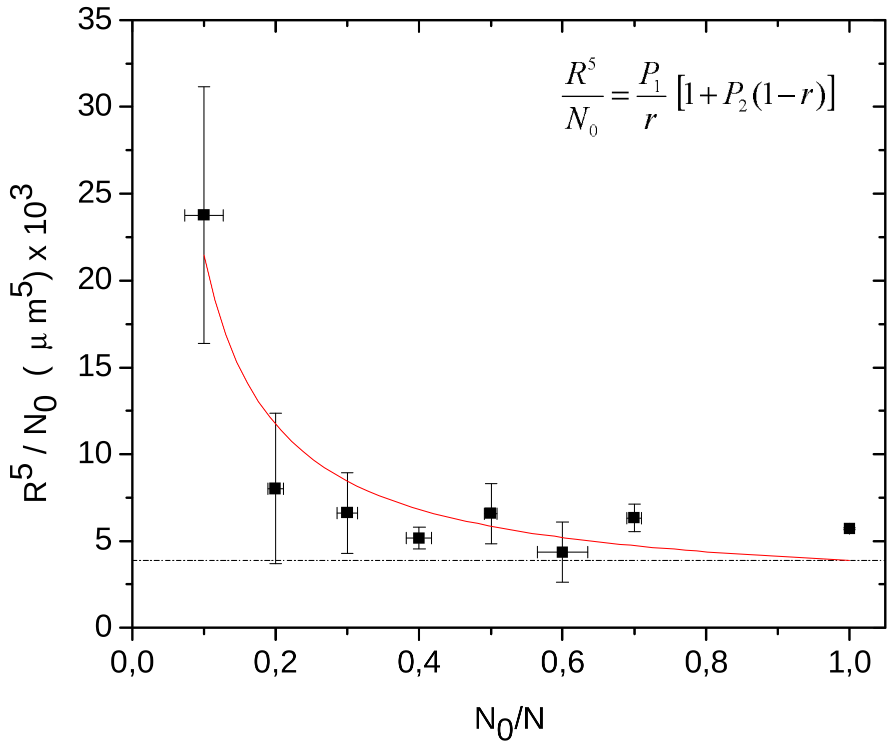
Hartree-Fock

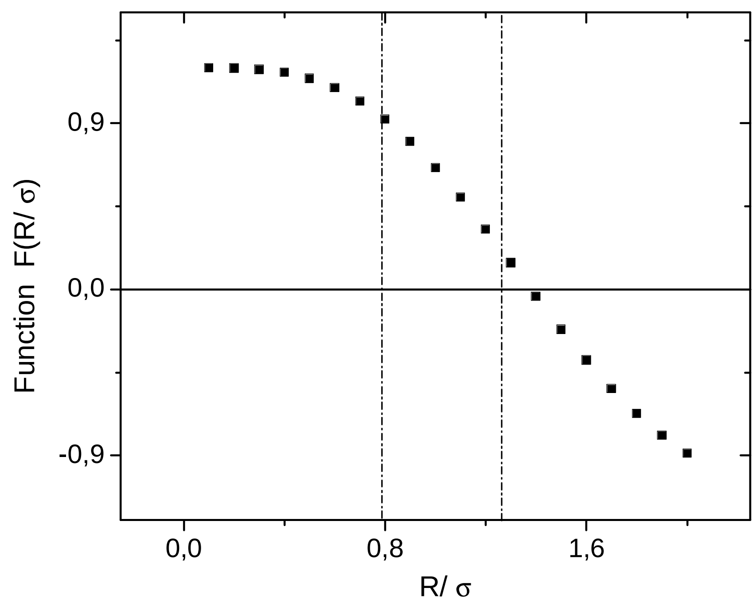
$$R^5 = \frac{15}{4\pi}\frac{U_0}{m\omega^2}\left[N - \frac{(c_0 U_0 f(R/\sigma) + c_T U_0 + c_L g(R/\sigma))T^3}{R^3}\right] \qquad c_L = \frac{8}{3\sqrt{\pi}}$$

$$\frac{R^5}{N_0} = \frac{15}{4\pi}\frac{U_0}{m\omega^2}\frac{1}{r}\left[1 - \frac{F(R/\sigma)}{N}\left(\frac{k_B}{\hbar\omega}\right)^3 (1-r)T_c^{\,3}\right] \qquad r = \frac{N_0}{N}$$

$$\Rightarrow \quad \boxed{\frac{R^5}{N_0} \approx \frac{15}{4\pi}\frac{U_0}{m\omega^2}\frac{1}{r}\left[1 - 0.83F(R/\sigma)(1-r)\right]}$$

Results





$$P_1 = C_{TOF} \qquad P_1 = (3900)$$

$$P_2 = -0.83\,F(R/\sigma) \qquad P_2 = (-0.49 \pm 0.05)$$

$$\left[\overline{R}/\sigma\right]_{in\,situ} \in [0.8\ ,\ 1.2]$$

$$F \sim 0.6$$

Experimental demonstration

Modulation of scattering length

Manipulating the scattering length

- Remember that the scattering length defines how strong the interaction is.
- So, tuning the interactions, effectively we are tuning the

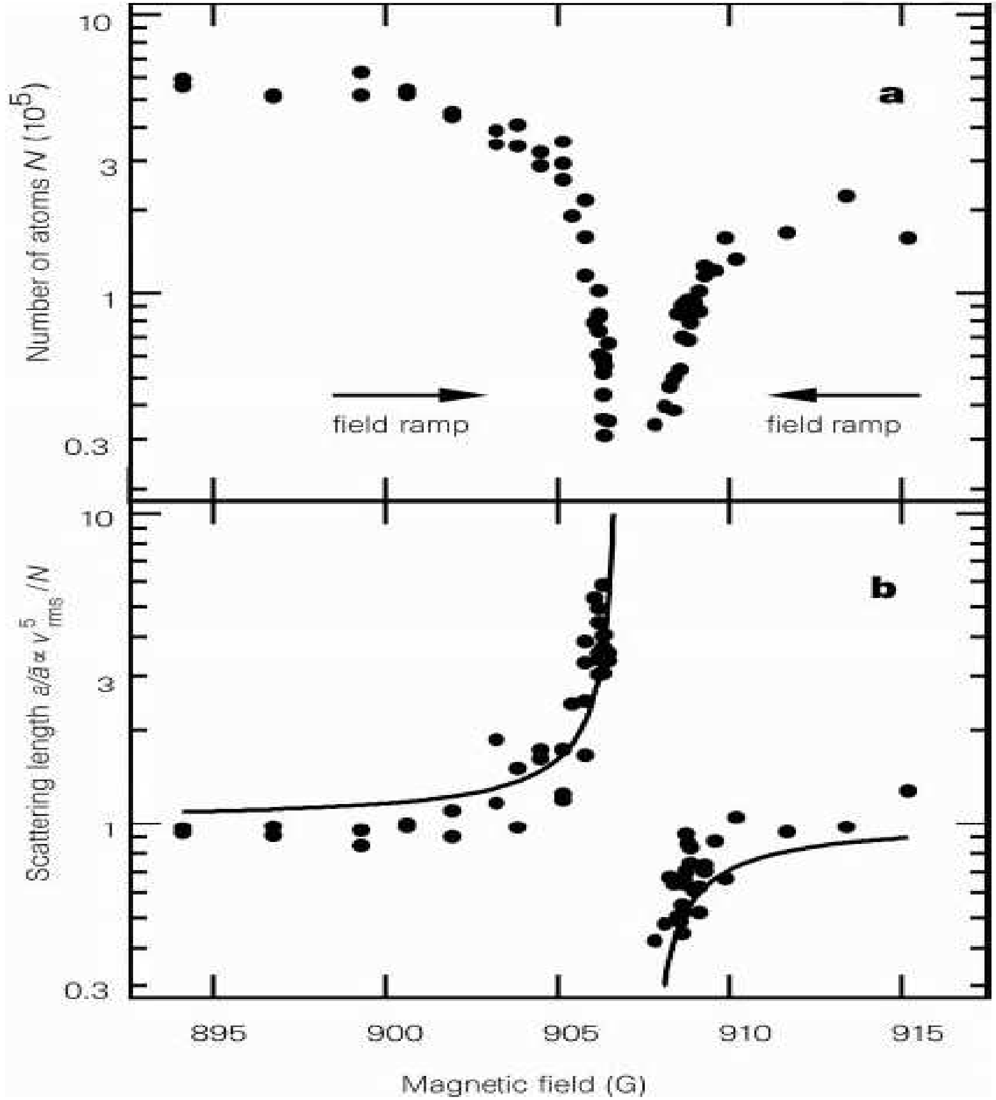
$$a_s = a_{nr}\left(1 - \frac{\Delta}{B - B_{res}}\right)$$

- How?

$$B_{res} = 907\,G \qquad \Delta = 1\,G$$

S. Inouye, *et al.*, Nature, **392**, 151 (1998)

For ^{23}Na



What will happen if...

- ...the magnetic field oscillates???

$$B(t) = B_0 + B\cos(\omega t)$$

$$\begin{aligned} a_s(t) &= a_{nr}\left(1 - \frac{\Delta}{B_0 - B_{res} + B\cos(\omega t)}\right) \\ &= a_{nr} - \frac{a_{nr}\Delta}{B_0 - B_{res}\left(1 - \frac{B}{B_0 - B_{res}}\cos(\omega t)\right)} \end{aligned}$$

$$\frac{B}{B_{res} - B_0} \ll 1 \Rightarrow a_s(t) \simeq \underbrace{a_{nr}\left(1 + \frac{\Delta}{B_{res} - B_0}\right)}_{a_0} + \underbrace{a_{nr}\frac{B\,\Delta}{(B_{res} - B_0)^2}}_{a} cos(\omega t)$$

Gross-Pitaevskii equation (GPE)

- Time-dependent scattering length

$$H\Phi(\vec{r},t) = i\hbar\frac{\partial\Phi(\vec{r},t)}{\partial t}$$

$$\begin{aligned} H(\Phi(\vec{r},t)) &= -\frac{\hbar^2}{2m}\nabla^2 + U_{trap}(\vec{r}) + \frac{4\pi\hbar^2 N}{m}a_s(t)|\Phi(\vec{r},t)|^2 \\ &= -\frac{\hbar^2}{2m}\nabla^2 + U_{trap}(\vec{r}) + \frac{4\pi\hbar^2 N}{m}a_0|\Phi(\vec{r},t)|^2 + \boxed{\frac{4\pi\hbar^2 N}{m}a|\Phi(\vec{r},t)|^2\cos(\omega t)} \end{aligned}$$

?

- GPE with an external oscillatory field

$$H(\Phi) = -\frac{\hbar^2}{2m}\nabla^2 + U_{trap}(\vec{r}) + \frac{4\pi\hbar^2 N}{m}a_s|\Phi|^2 + \boxed{V(\vec{r})\cos(\omega t)}$$

Formation of topological modes

Modulation of scatt. Length and excitation of collective modes

– Dipolar modes

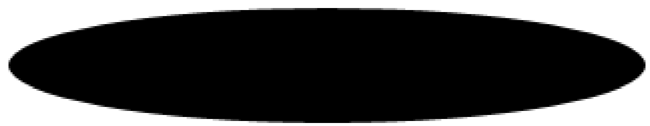
– Breathing modes

– Scissors modes

– Quadrupole modes

Introduction

– Dipolar modes



Introduction

– Breathing modes

Introduction

– Scissors modes

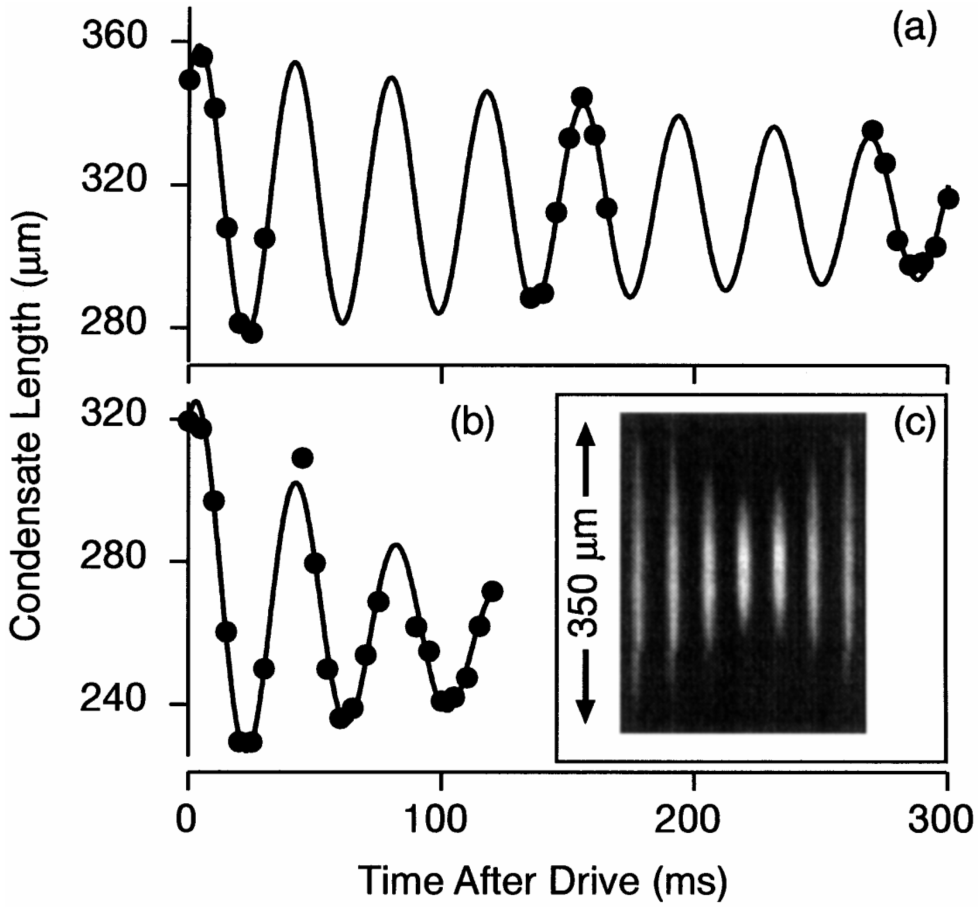
Introduction

– Quadrupole modes

- Experimental observation of quadrupolar modes

How?

Modulating trap frequencies



D. M. Stamper-Kurn, *et al. Phys. Rev. Lett.* **81**, 500 - 503 (1998)

Introduction

– We modulated the scattering length applying a time-dependent magnetic field close to a Feshbach resonance.

– The question is:

  - Would be possible excite a BEC modulating the interatomic interaction?

Experimental Details @ RICE

Trap

- $N = 3 \times 10^5$ ^{7}Li atoms
- $|F=1, m_F = 1>$
- $\omega_r = 2\pi \times 255$ Hz and $\omega_z = 2\pi \times 5.5$ Hz

Excitation

- $B(t) = B_0 + B\cos(\omega t)$, $B_0 = 565$ G, $B = 0.018\, B_0$

Excitation during t_{exc} | Free oscillation | In situ image

- $a_s \approx a_{so} + a\cos(\omega t)$, $a_{s0} = 3a_0$, $a = 1.6a_0$

Theory: Variational Method

- New dimensioless variables

$$w_\eta = l \; u_\eta \qquad l = \sqrt{\frac{\hbar}{m\omega_r}}$$

- Time evolution of parameters

$$\ddot{u}_r + u_r = \frac{1}{u_r^3} + \frac{1}{u_r^3 u_z}\sqrt{\frac{2}{\pi}}\left[\frac{N a_s}{l}\right]$$

$$\ddot{u}_z + \lambda^2 u_z = \frac{1}{u_z^3} + \frac{1}{u_r^2 u_z^2}\sqrt{\frac{2}{\pi}}\left[\frac{N a_s}{l}\right]$$

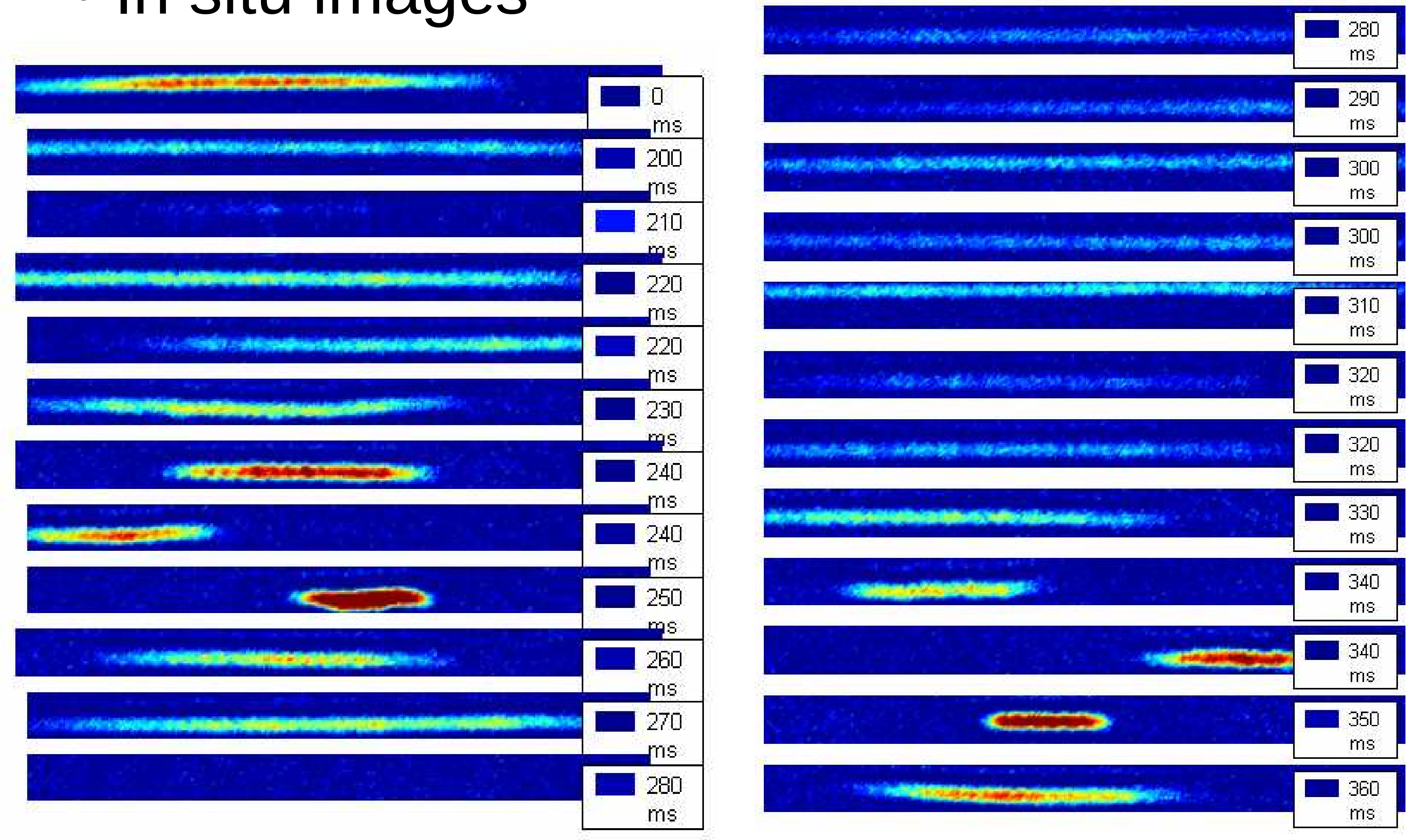
$$u_\eta = \sqrt{\frac{2}{7}} R_\eta$$

- Relation with … radius

Results

$\omega = 2\pi \times 10$ Hz , $t_{exc} =$ 260ms

- In situ images



Theory: Variational Method

- Trial function

$$\psi(x,y,z,t) = A(t)\ \Pi_{\eta=x,y,z} e^{-\frac{\eta^2}{2w_\eta^2(t)}} e^{-i\eta^2\beta_\eta(t)}$$

- Lagrangian density

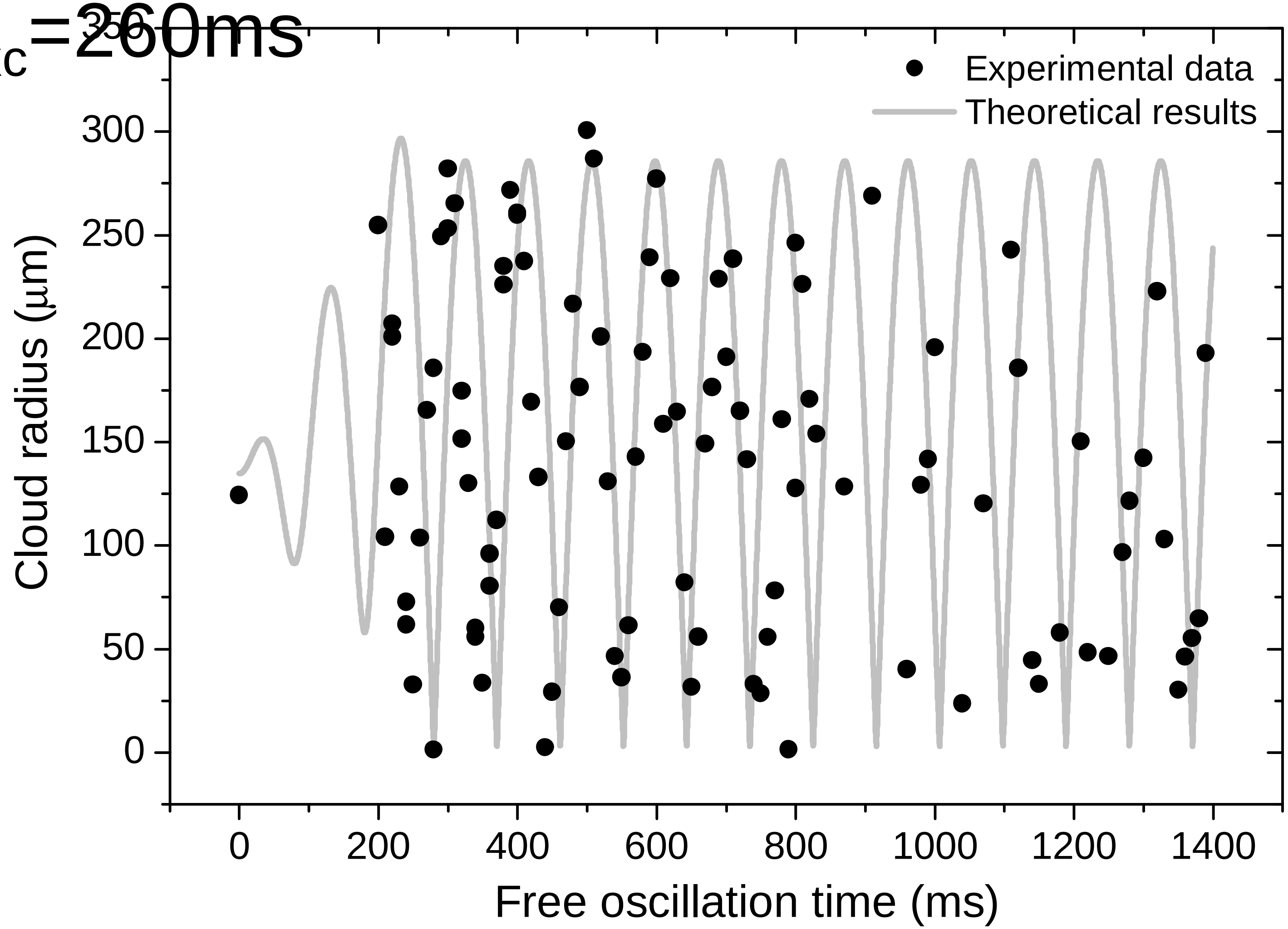
$$\ell = \frac{i\hbar}{2}\left(\psi(\vec{r})\frac{\partial\psi(\vec{r})^*}{\partial t} - \psi(\vec{r})^*\frac{\partial\psi(\vec{r})}{\partial t}\right) + \frac{\hbar^2}{2m}|\nabla\psi(\vec{r})|^2 + V(\vec{r})|\psi(\vec{r})|^2 + \frac{2\pi a_s\hbar^2}{m}|\psi(\vec{r})|^4$$

- Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \qquad L = \langle\ell\rangle = \int \ell\ d^3r$$

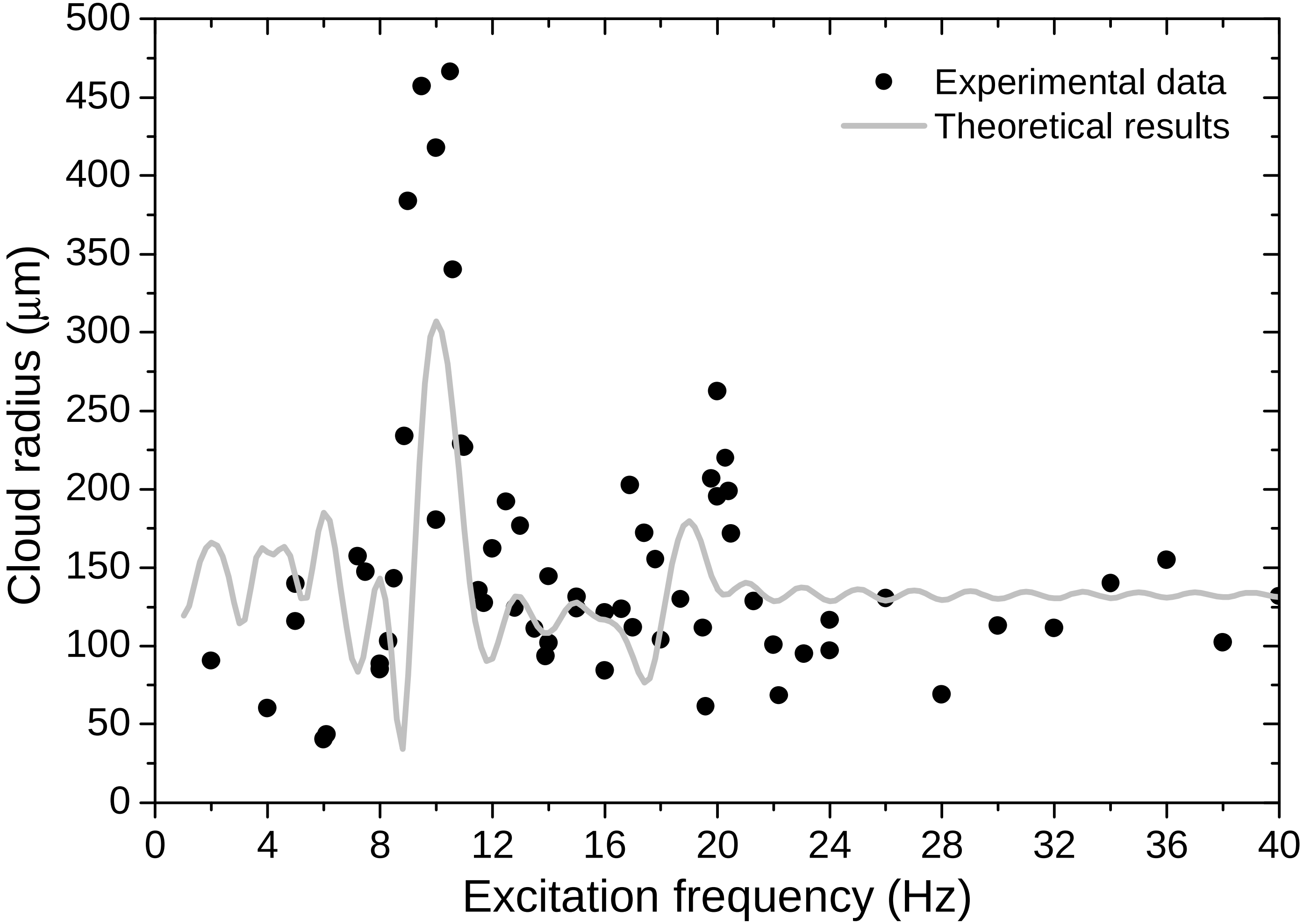
Results

- Quadrupolar oscilations: $\omega=2\pi\times10Hz$, t_{exc}=260ms



Results

- Axial cloud radius after t_{exc} = 500ms, no free oscillation time



Time-Dependent Perturbation Theory

- GPE

$$H(\Phi(\vec{r},t)) = \underbrace{-\frac{\hbar^2}{2m}\nabla^2 + U_{trap}(\vec{r}) + A_0|\Phi(\vec{r},t)|^2}_{H_0} + \underbrace{A|\Phi(\vec{r},t)|^2\cos(\omega t)}_{V}$$

perturbation

$$A_0 = \frac{4\pi\hbar^2 N}{m}a_0 \qquad A = \frac{4\pi\hbar^2 N}{m}a$$

- Consider that the solutions of

$$\int \phi_m^* \left((H+V)\,\Phi = i\hbar\frac{\partial\Phi}{\partial t}\right)_m^* i\hbar\frac{\partial\Phi}{\partial t}d\vec{r}$$

are given by

$$\Phi = \sum_n c_n(t)\phi_n(\vec{r})e^{-iE_nt/\hbar}$$

$$n_i(t) = |c_i(t)|^2$$

Coefficients equations

- The equation

$$
\begin{aligned}
i\hbar\frac{dc_m}{dt} &= A_0 \sum_{n,k\neq n} |c_k|^2 c_n e^{i\omega_{mn}t} \int \phi_m^* \left(|\phi_k|^2 - |\phi_n|^2\right) \phi_n d\vec{r} \\
&+ A_0 \sum_{n',k',l'\neq k'} c_{k'}^* c_{l'} c_{n'} e^{i(\omega_{mn'}+\omega_{k'l'})t} \int \phi_m^* \phi_{k'}^* \phi_{l'} \phi_{n'} d\vec{r} \\
&+ A \sum_{\tilde{n},\tilde{k},\tilde{l}} c_{\tilde{k}}^* c_{\tilde{l}} c_{\tilde{n}} e^{i(\omega_{m\tilde{n}}+\omega_{\tilde{k}\tilde{l}})t} \cos(\omega t) \int \phi_m^* \phi_{\tilde{k}}^* \phi_{\tilde{l}} \phi_{\tilde{n}} d\vec{r}
\end{aligned}
$$

- It is hard to work with because
  - A lot of terms

Approximation

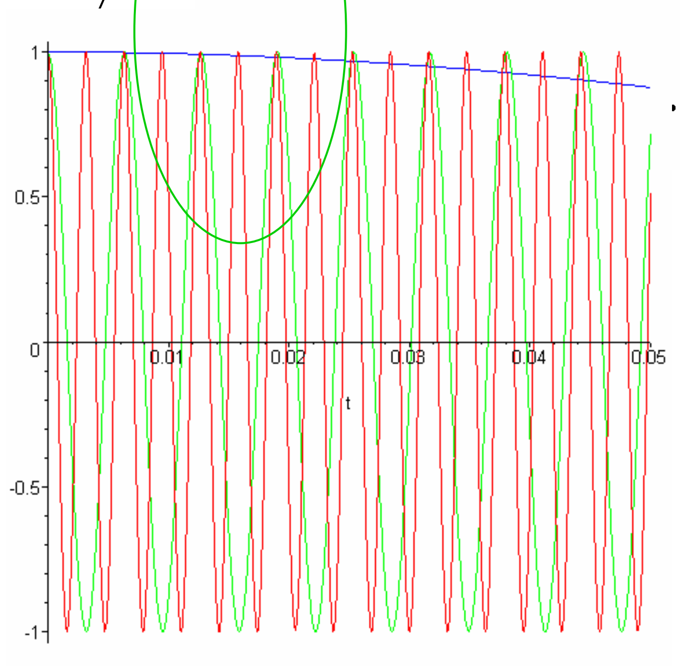
- Time average technique: if $\Delta\omega = \omega - \omega_{p0} \rightarrow 0$

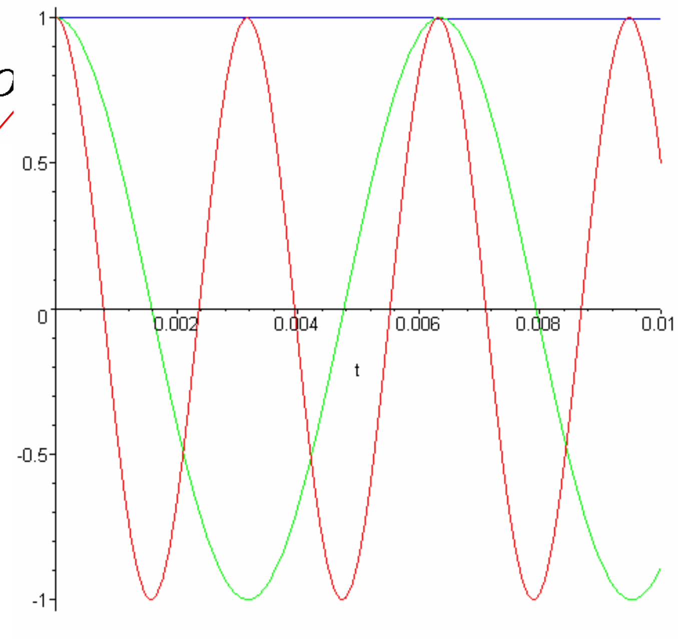
≈ constant ≈ constant

$$\frac{dc_0}{dt} = \cdots + \cdots \cos\omega t + \cdots e^{i(\omega-\omega_{p0})t} + \cdots e^{i(\omega+\omega_{p0})t}$$

$\cdots co$ … $\cdots e^{-i(\omega+\omega_{p0})t}$

Just check: ω=990 Hz and ω_{p0}=1000 Hz





$\cos(\omega_{mn}t)$
$\cos(\omega+\omega_{mn})t$
$\cos(\Delta\omega t)$

Two-level system

- If there is no combination between two frequencies that summed or subtracted

$$
\begin{aligned}
i\hbar\frac{dc_m}{dt} &= A_0 \sum_{n,k\neq n} |c_k|^2 c_n e^{i\omega_{mn}t} \int \phi_m^* \left(|\phi_k|^2 - |\phi_n|^2\right) \phi_n d\vec{r} \\
&+ A_0 \sum_{n',k',l'\neq k'} c_{k'}^* c_{l'} c_{n'} e^{i(\omega_{mn'}+\omega_{k'l'})t} \int \phi_m^* \phi_{k'}^* \phi_{l'} \phi_{n'} d\vec{r} \\
&+ A \sum_{\tilde{n},\tilde{k},\tilde{l}} c_{\tilde{k}}^* c_{\tilde{l}} c_{\tilde{n}} e^{i(\omega_{m\tilde{n}}+\omega_{\tilde{k}\tilde{l}})t} \cos(\omega t) \int \phi_m^* \phi_{\tilde{k}}^* \phi_{\tilde{l}} \phi_{\tilde{n}} d\vec{r}
\end{aligned}
$$

- We can assume that a resonant oscillation of the interaction gives us a two level

Coefficients equations

- After some calculations, we easily obtain

$$\begin{aligned}\frac{dc_0}{dt} &= -\frac{iA_0}{\hbar}\left|c_p\right|^2 c_0 \left(2I_{0,p,0} - I_{0,0,0}\right) \\ &\quad -\frac{iA}{2\hbar}e^{i\Delta\omega t}\left[\left|c_p\right|^2 c_p I_{0,p,p} + 2\left|c_0\right|^2 c_p I_{0,0,p} + c_p^* c_0^2 I_{p,0,0} e^{-2i\Delta\omega t}\right]\end{aligned}$$

$$\begin{aligned}\frac{dc_p}{dt} &= -\frac{iA_0}{\hbar}\left|c_0\right|^2 c_p \left(2I_{p,0,p} - I_{p,p,p}\right) \\ &\quad -\frac{iA}{2\hbar}e^{-i\Delta\omega t}\left[\left|c_0\right|^2 c_0 I_{p,0,0} + 2\left|c_p\right|^2 c_0 I_{p,p,0} + c_0^* c_p^2 I_{0,p,p} e^{2i\Delta\omega t}\right]\end{aligned}$$

where

$$I_{n,k,l} = \int \phi_n^* |\phi_k|^2 \phi_l d\vec{r}$$

Normalized equations

- Some details before the graphs

$$U_{trap} = \frac{m}{2}(\omega_r^2 r^2 + \omega_z^2 z^2) \qquad \lambda = \frac{\omega_z}{\omega_r}$$

$$l_r = \sqrt{\frac{\hbar}{m\omega_r}} \qquad g_0 = 4\pi N \frac{a_0}{l_r} \qquad \psi(\vec{x}) = l_r^{3/2}\phi(\vec{r})$$

$$x_r = \frac{r}{l_r} \qquad x_z = \frac{z}{l_r} \qquad H(\psi) = \frac{H(\phi)}{\hbar\omega_r}$$

$$t' = \omega_r t \qquad \delta = \frac{\Delta\omega}{\omega_r}$$

$$\Rightarrow \frac{A_0}{\hbar} \underbrace{\int \phi_n^* |\phi_k|^2 \phi_l d\vec{r}}_{I_{n,k,l}} = g_0 \omega_r \underbrace{\int \psi_n^* |\psi_k|^2 \psi_l d\vec{x}}_{J_{n,k,l}}$$

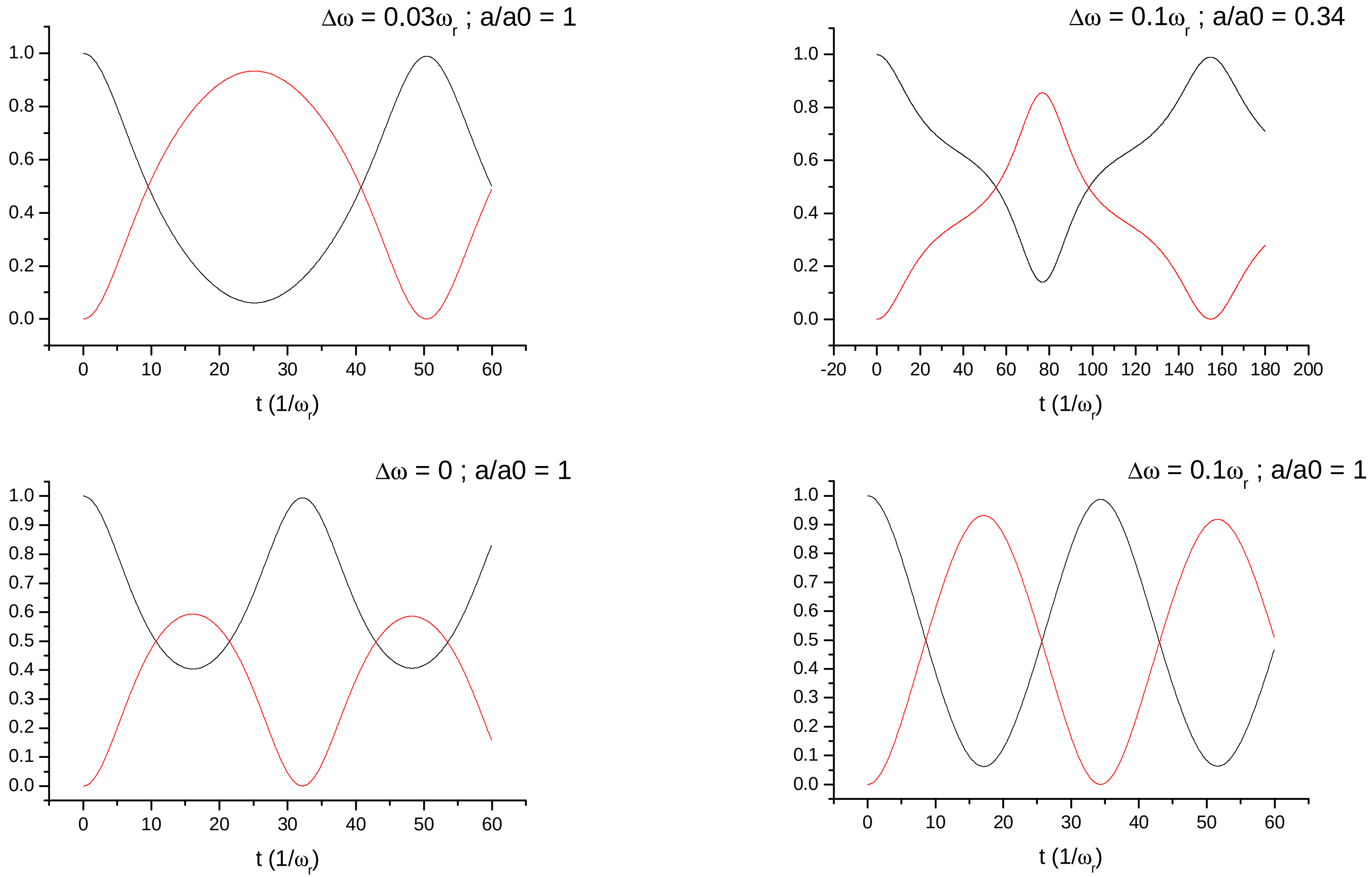
Normalized equations

- The final equations

$$\frac{dc_0}{dt} = -ig_0 \left|c_p\right|^2 c_0 \left(2J_{0,p,0} - J_{0,0,0}\right) - \frac{ig_0}{2} e^{i\delta t'} \frac{a}{a_0} \left[\left|c_p\right|^2 c_p J_{0,p,p} + +2 \left|c_0\right|^2 c_p J_{0,0,p} + c_p^* c_0^2 J_{p,0,0} e^{-2i\delta t'} \right]$$

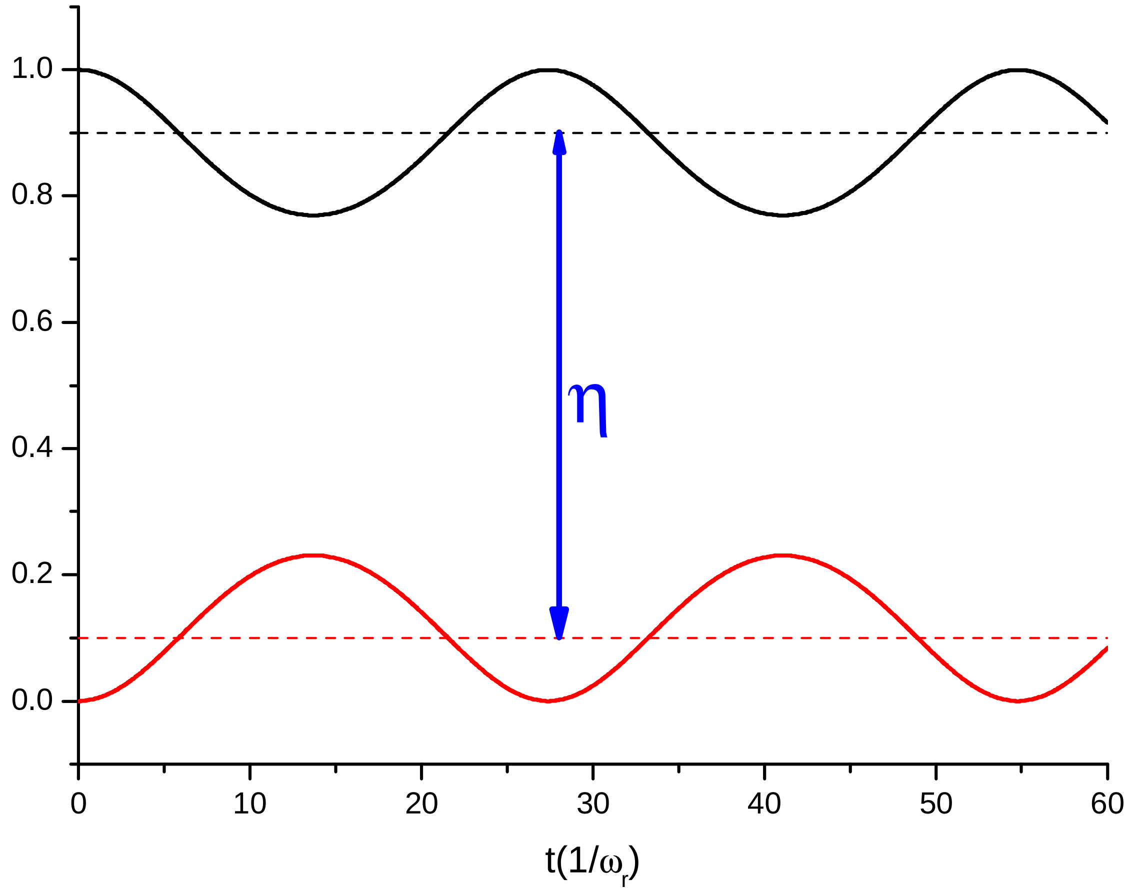
$$\frac{dc_p}{dt} = -ig_0 \left|c_0\right|^2 c_p \left(2J_{p,0,p} - J_{p,p,p}\right) - \frac{ig_0}{2} e^{-i\delta t'} \frac{a}{a_0} \left[\left|c_0\right|^2 c_0 J_{p,0,0} + +2 \left|c_p\right|^2 c_0 J_{p,p,0} + c_0^* c_p^2 J_{0,p,p} e^{2i\delta t'} \right]$$

Temporal evolution of populations



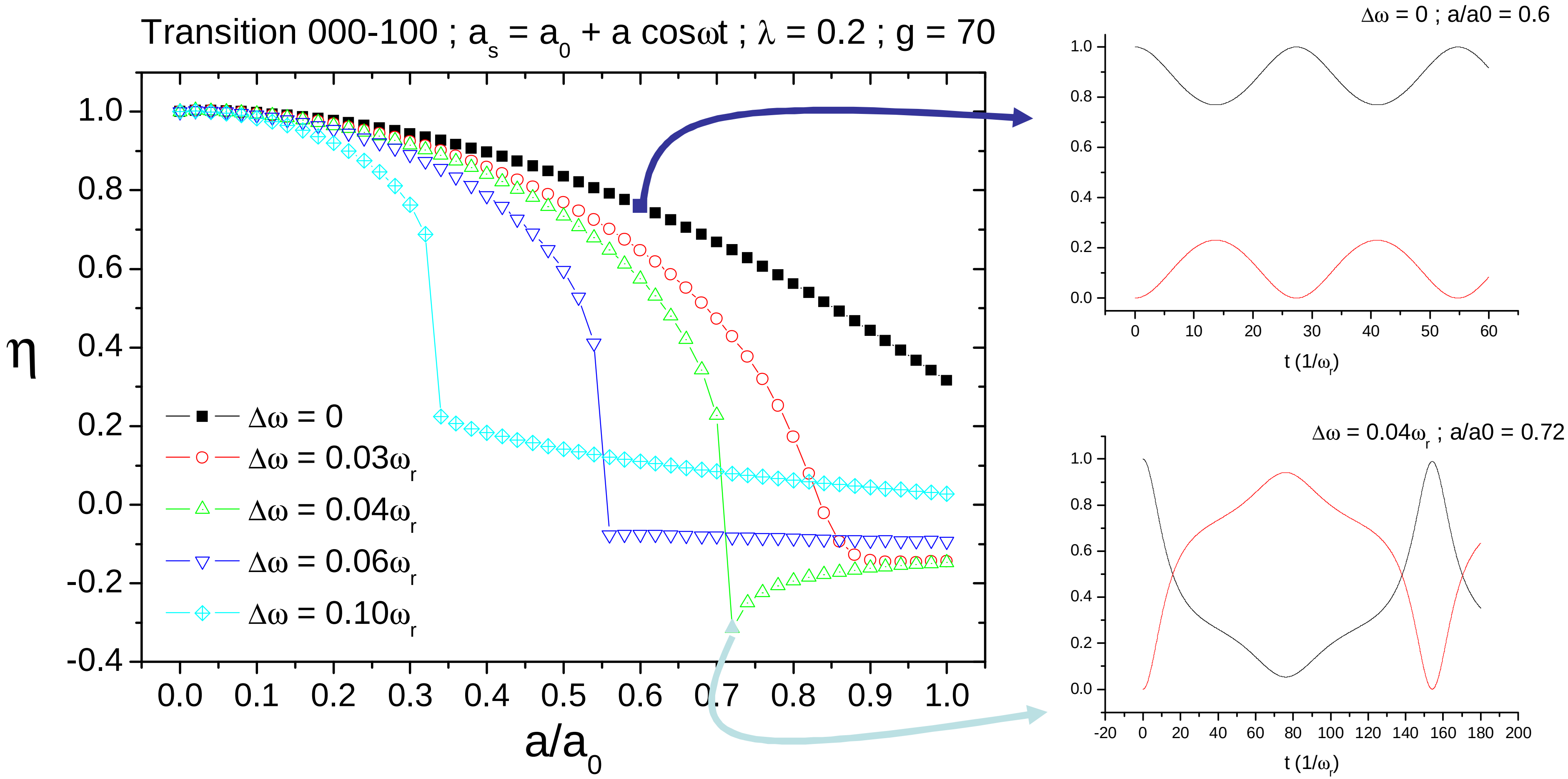
Order parameter

- Definition: $\eta = \bar{n}_0 - \bar{n}_p$



Order Parameter

- Results



Real world

- Remember that

$$B(t) = B_0 + Bcos(\omega t) \qquad a_s(t) = a_{bg}\left(1 - \frac{\Delta}{B(t) - B_{res}}\right)$$

$$\frac{B}{B_{res} - B_0} \ll 1 \Rightarrow a_s \simeq \underbrace{a_{nr}\left(1 + \frac{\Delta}{B_{res} - B_0}\right)}_{a_0} + \underbrace{a_{nr}\frac{B\,\Delta}{(B_{res} - B_0)^2}}_{a} cos(\omega t)$$

$$g_0 = \frac{4\pi N a_0}{l_r} \qquad \Rightarrow \qquad N = \frac{g_0\sqrt{\frac{\hbar}{m_0\omega_r}}}{4\pi a_{nr}\left(1 + \frac{\Delta}{B_r - B_0}\right)} \qquad l_r = \sqrt{\frac{\hbar}{m_0\,\omega_r}}$$

- Taking these values

$$g_0 = 70 \qquad \omega_r = 2\pi \times 120Hz \qquad \frac{B}{B_{res} - B_0} = \begin{cases} 0.1, & {}^{85}Rb,\ {}^{7}Li,\ {}^{39}K \\ -0.1, & {}^{87}Rb \end{cases}$$

Real world: numerical values

$$\frac{a}{a_0} = 0.2$$

Atom	$B_{res}(G)$	$\Delta(G)$	$a_{bg}(a_{Bohr})$	$B_0(G)$	$B(G)$	$a_0(a_{Bohr})$	$a(a_{Bohr})$	N
^{85}Rb	155.0	10.7	-443	160.4	-0.54	443	88.6	237
^{87}Rb	1007.34	0.17	100	1007.6	0.026	33.3	6.7	3109
^{7}Li	735	-113	-27.5	678.5	5.65	27.5	5.5	13263
^{39}K	403.4	-52	-23	377.4	2.6	23	4.6	6729

Real world: numerical values

$$\frac{a}{a_0} = 0.8$$

Atom	$B_{res}(G)$	$\Delta(G)$	$a_{bg}(a_{Bohr})$	$B_0(G)$	$B(G)$	$a_0(a_{Bohr})$	$a(a_{Bohr})$	N
^{85}Rb	155.0	10.7	-443	164.4	-0.9	63	51	1657
^{87}Rb	1007.34	0.17	100	1007.53	0.02	11	9	9327
^{7}Li	735	-113	-27.5	636	10	3.9	3.1	92841
^{39}K	403.4	-52	-23	357.9	4.55	3.3	2.6	47104

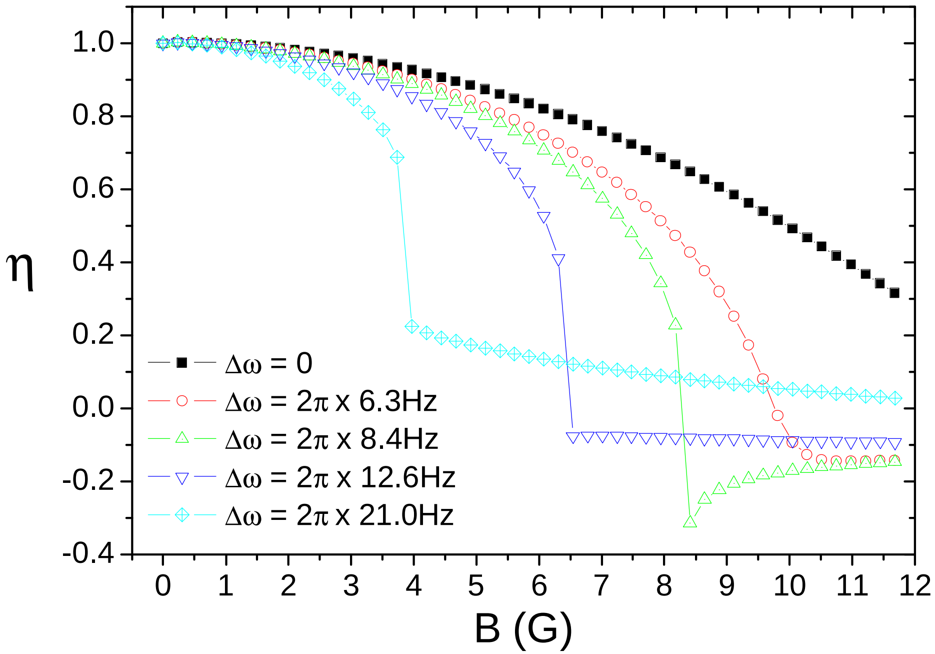
Chosen Atom #1: ^{7}Li

$m_0 = 11.65 \times 10^{-24} g \qquad a_{nr} = -27.5 a_{Bohr}$

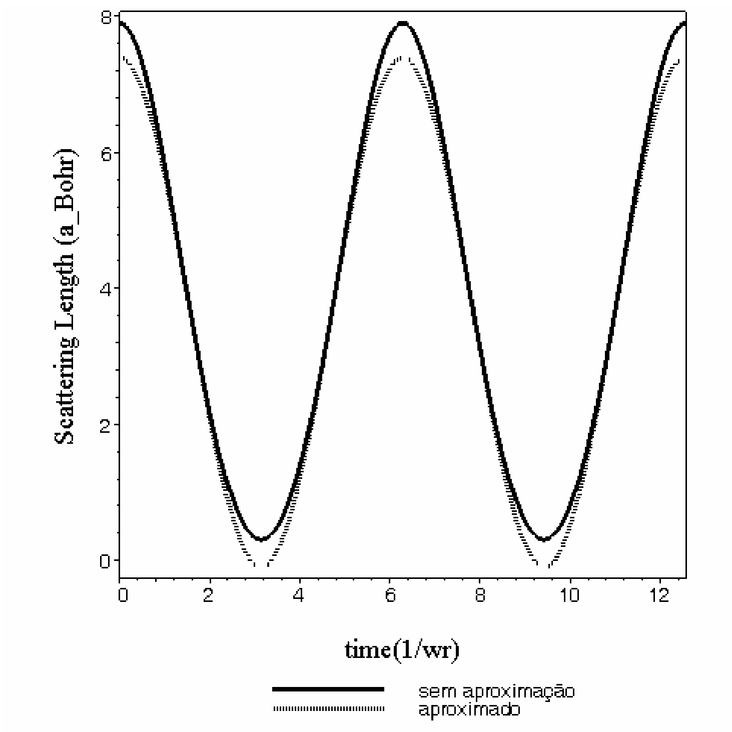
$B_{res} = 735G \qquad \Delta = -113G$

$$\omega_{100} = \frac{E_{100} - E_0}{\hbar} = 2\pi \times 209.6Hz$$

$N = 100,000 \qquad B_0 = 635.2G$

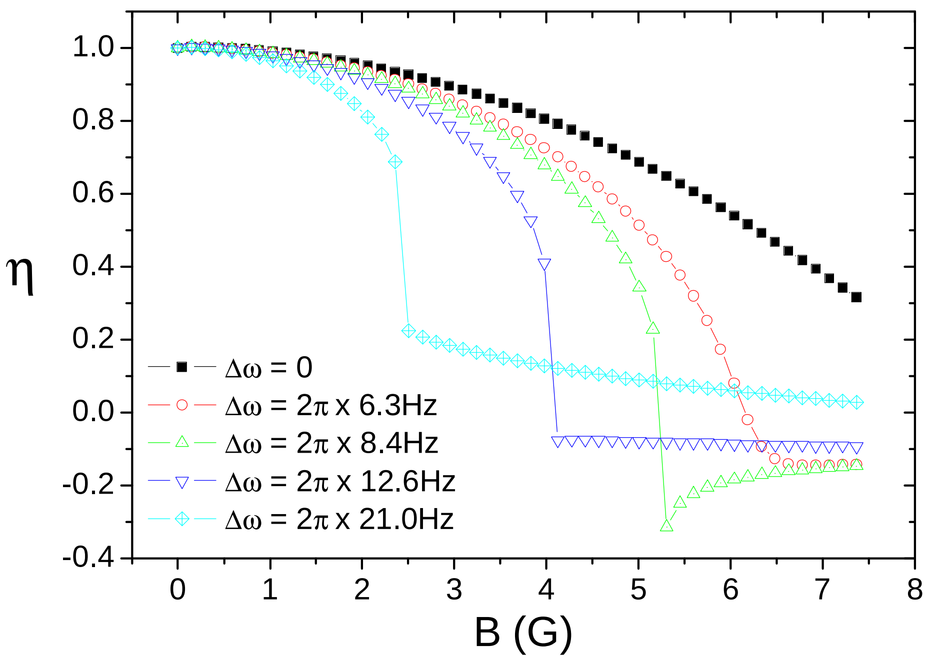


$B = 12G$

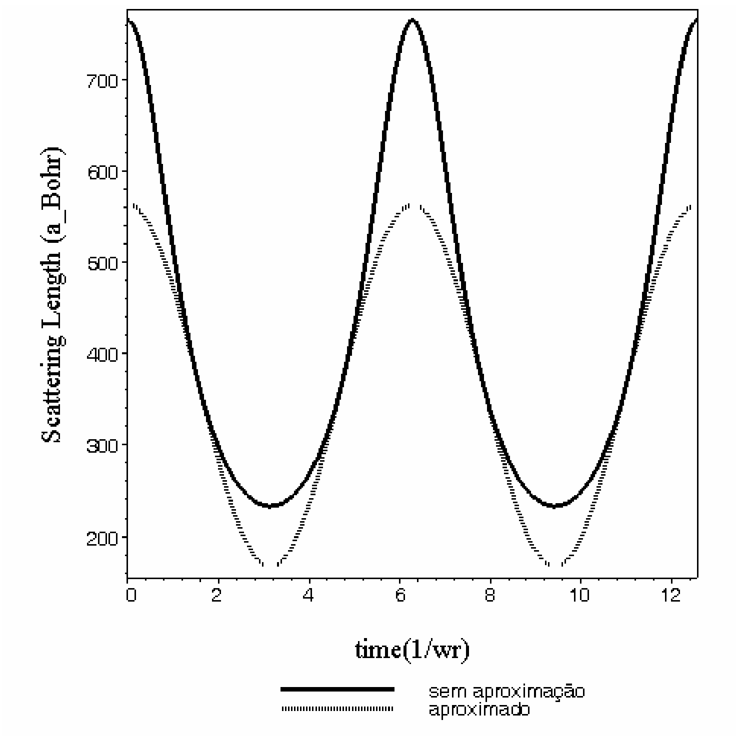


Chosen Atom #1: ^{7}Li

$N = 1,000 \qquad B_0 = 727.1G$



$B = 4G$



Chosen Atom #2: ^{39}K

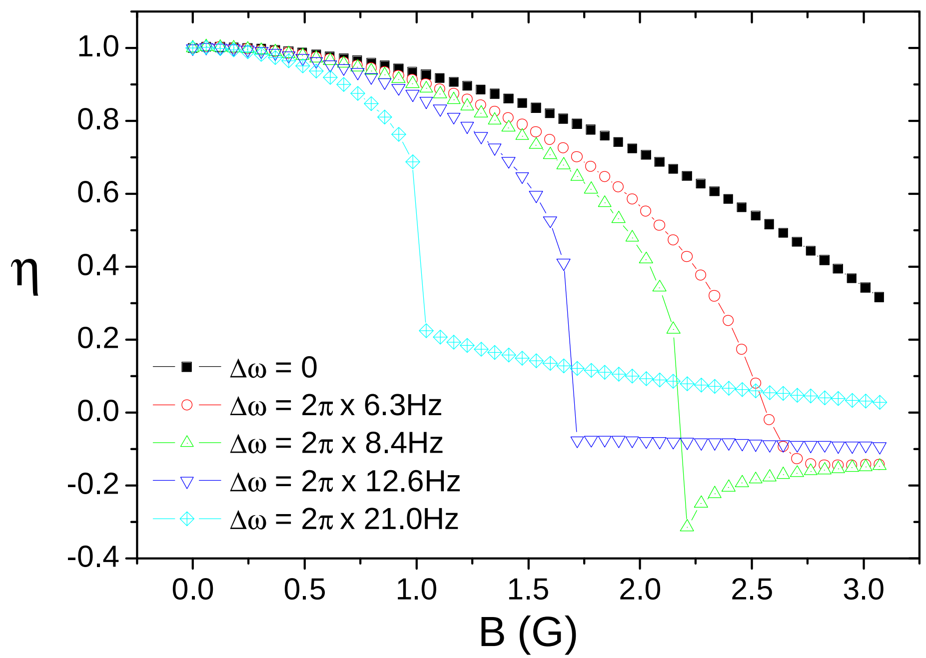
$$m_0 = 64.70 \times 10^{-24} g \qquad a_{bg} = -23 a_{Bohr}$$

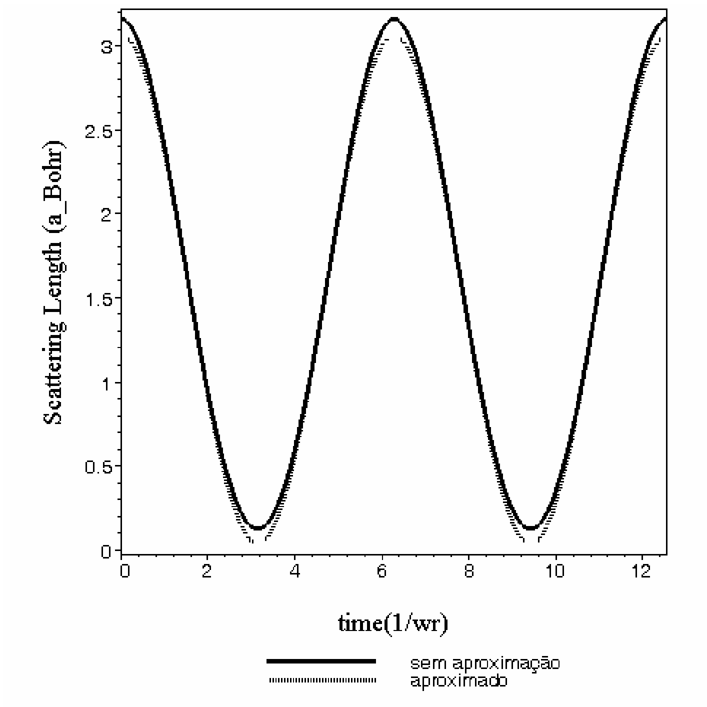
$$B_{res} = 403.4G \qquad \Delta = -52G$$

$$\omega_{100} = \frac{E_{100} - E_0}{\hbar} = 2\pi \times 209.6Hz$$

$$N = 100,000 \qquad B_0 = 354.7G$$

$$B = 3G$$

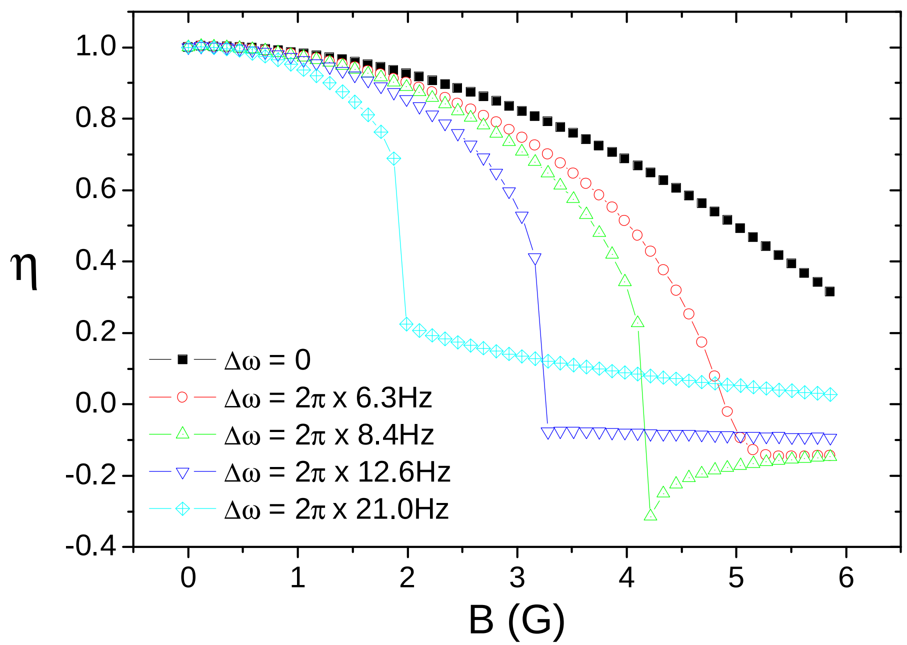


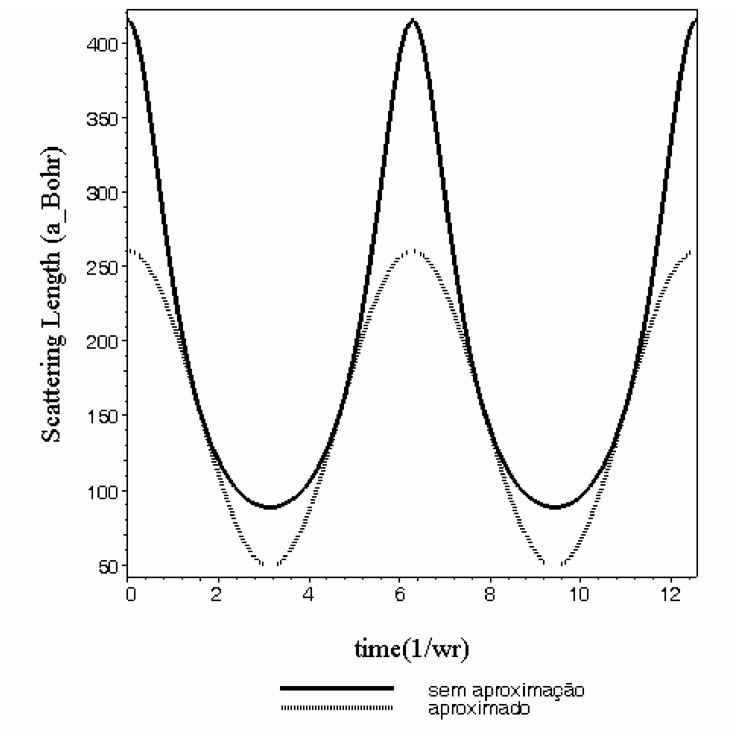


Chosen Atom #2: ^{39}K

$N = 1,000 \qquad B_0 = 396.7G$

$B = 4G$





Experimental Details @ RICE

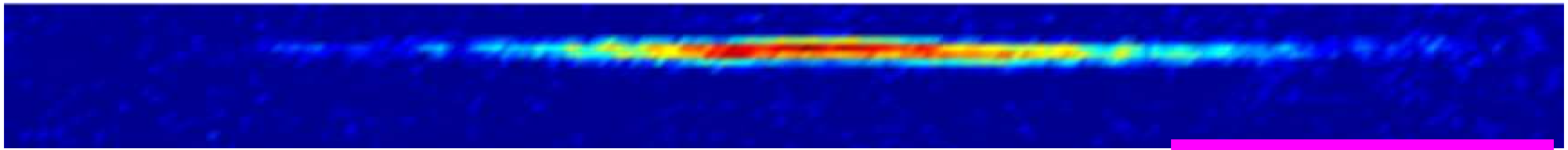
Trap

- $N = 3 \times 10^5$ ^{7}Li atoms
- $|F=1 , m_F =1 >$
- $\omega_r = 2\pi \times 255$ Hz and $\omega_z = 2\pi \times 5.5$ Hz

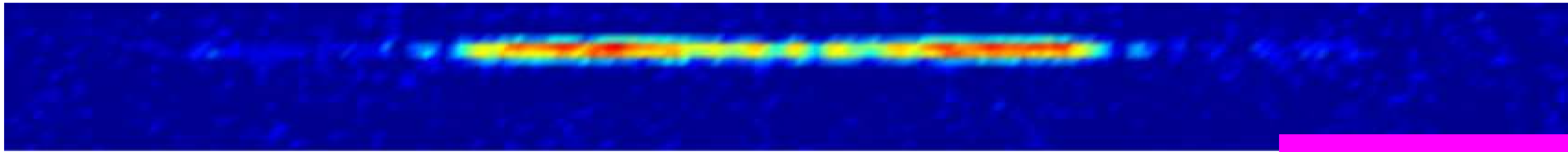
Excitation

- $B(t)= B_0 + B \cos(\omega t)$, $B_0 = 565$ G , $B = 0.018$
  $a_s \approx a_{so} + a \cos(\omega t)$, $a_{s0} = 3a_0$, $a = 1.6a_0$

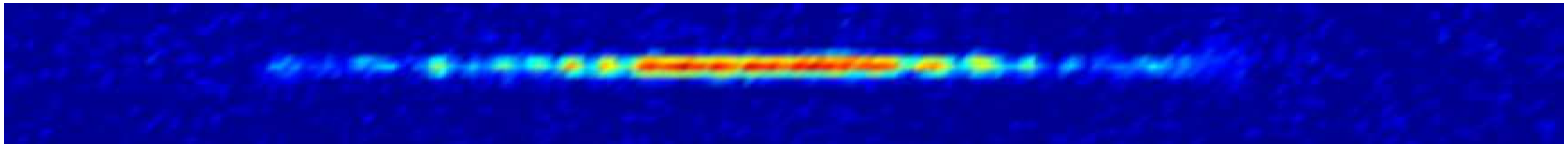
xcitation during t_{exc} | Free oscillation | In situ image



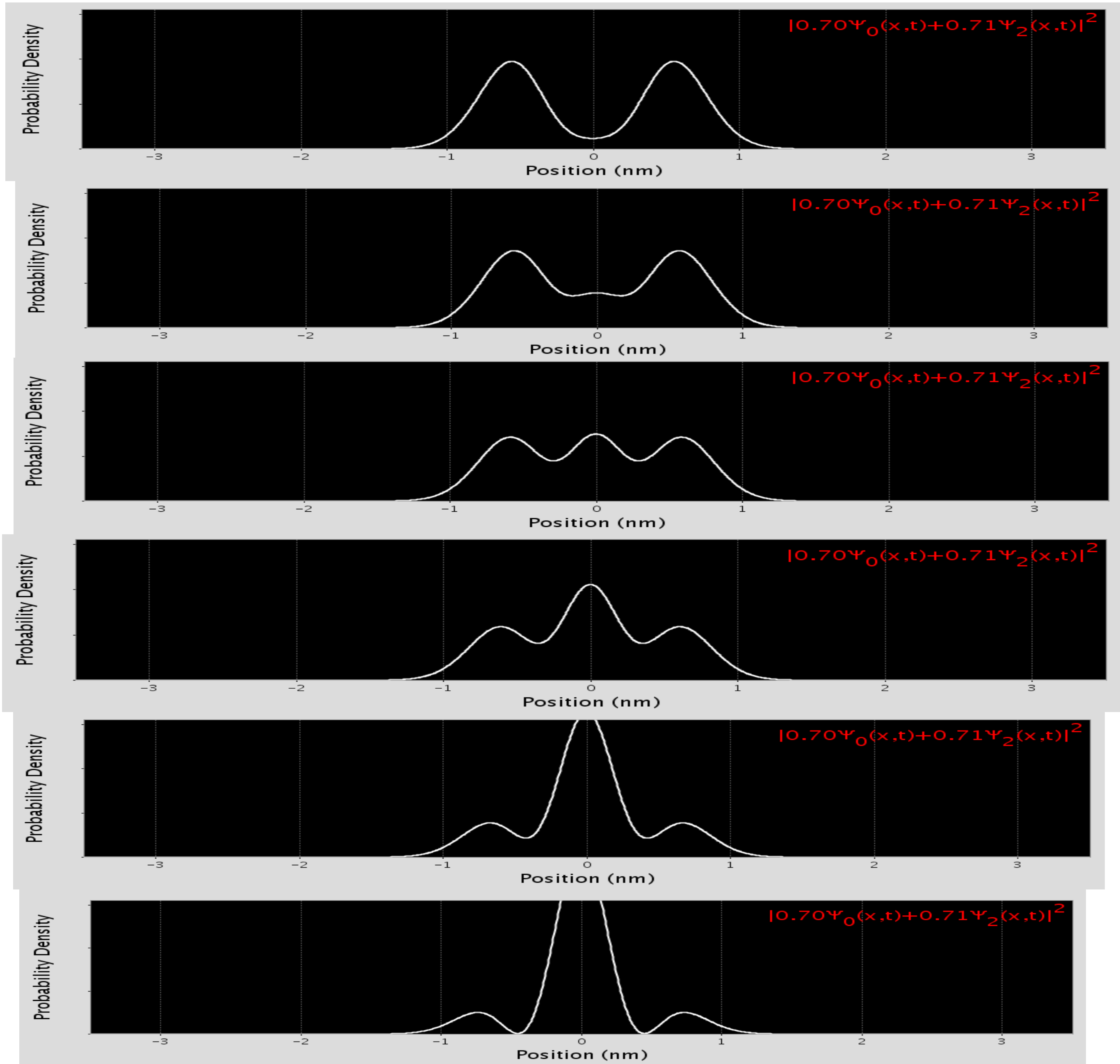
30Hz 600ms



30Hz 800ms

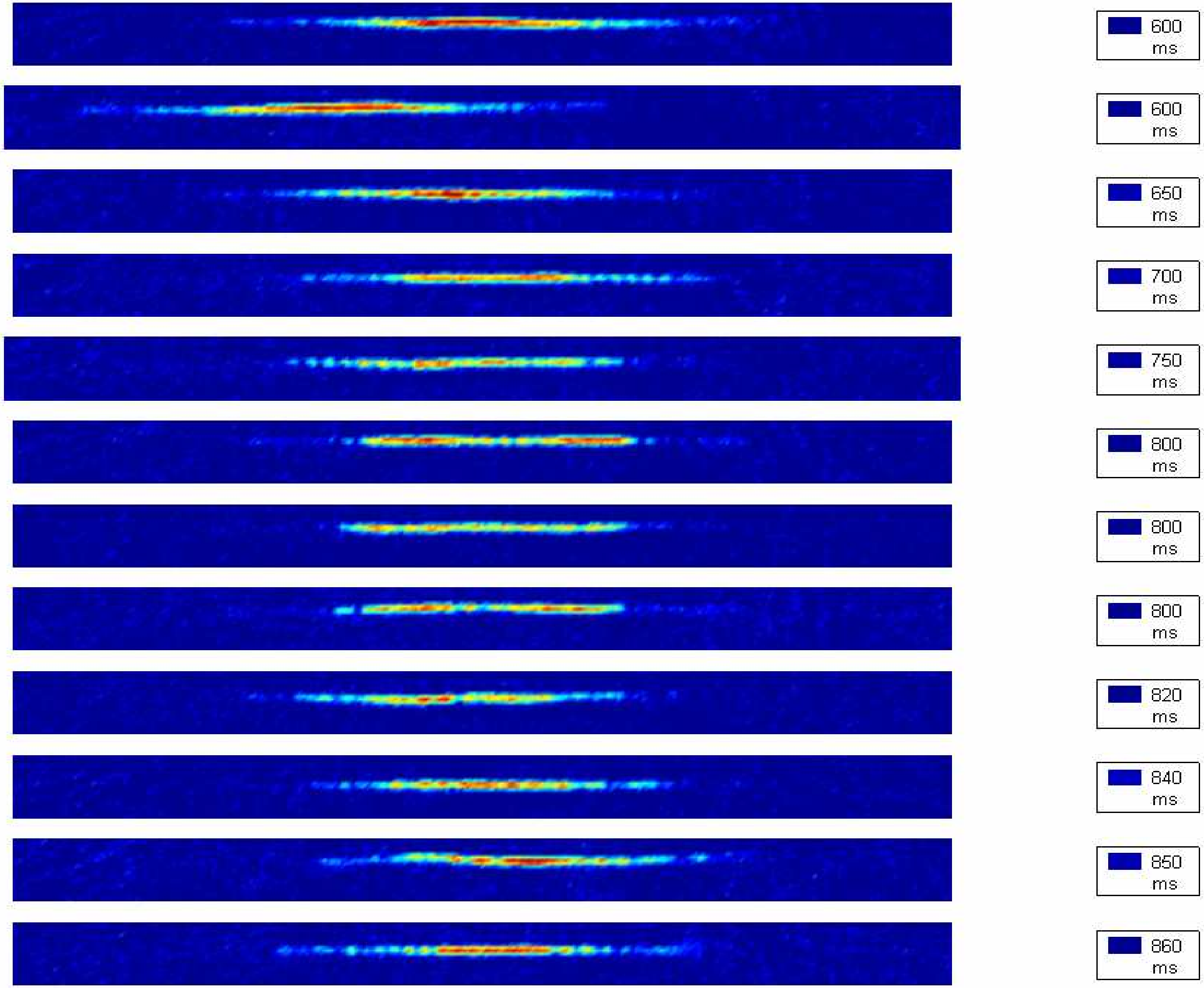


30Hz 860ms



Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)
Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)
Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)
Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)
Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)
Probability Density
|0.70Ψ0(x,t)+0.71Ψ2(x,t)|2
−3 −2 −1 0 1 2 3
Position (nm)

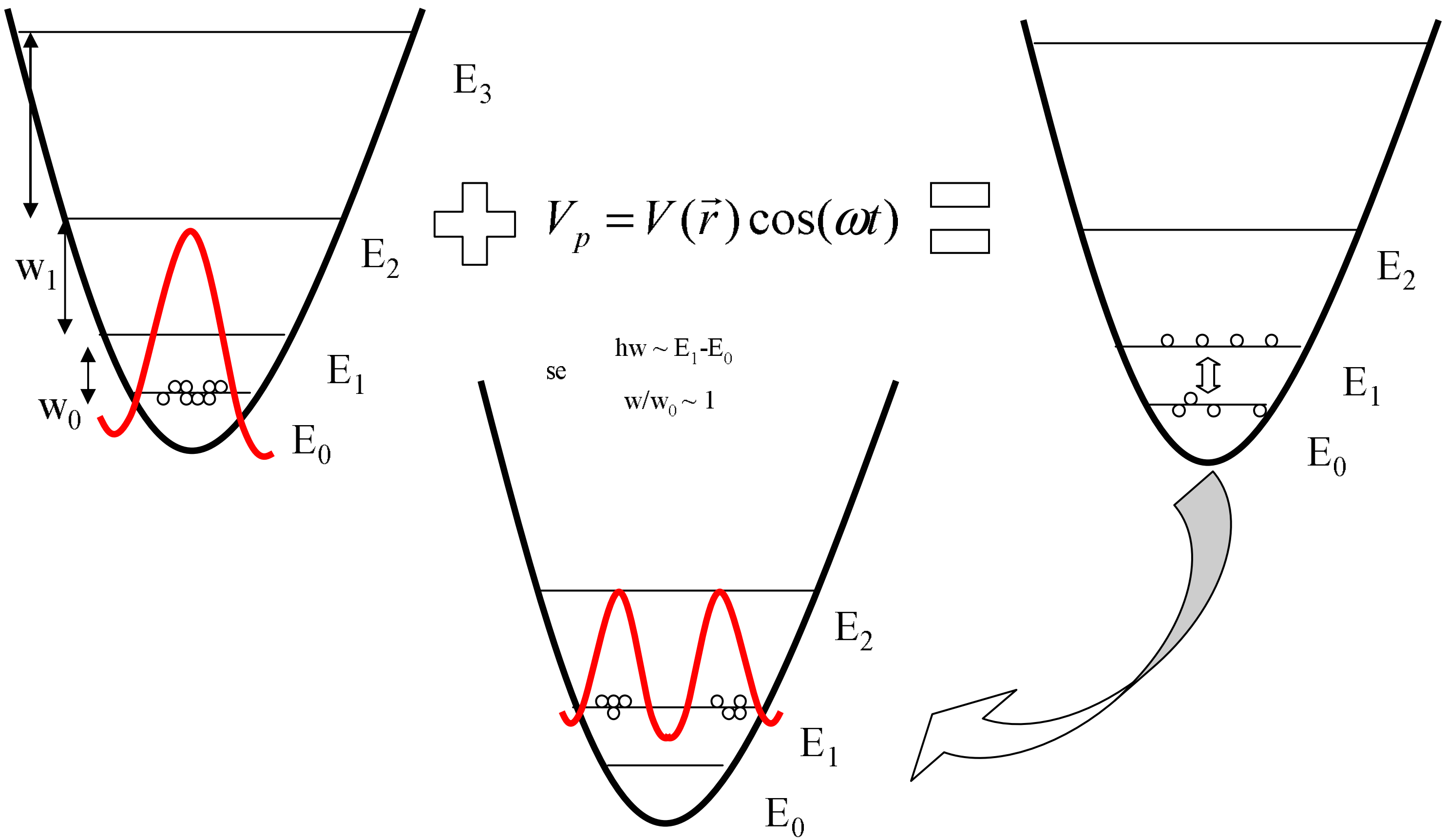
Mod_30Hza.dat images:8194-8226



Simulation for the time evolution

EXCITATION OF COHERENT MODES

Coherent population transfer between two trapped states



INTERACTIONS

$$H(\mathbf{p},\mathbf{r}) = \frac{p^2}{2M} + U(\mathbf{r}) + \gamma n(\mathbf{r})$$

$$\rightarrow \quad \frac{N_0}{N} = 1 - \left(\frac{T}{T_c^0} \right)^{\eta+1} - \gamma A \left(\frac{T}{T_c^0} \right)^{\eta+3/2}$$

Gross-Pitaevskii equation

$$H(\varphi) = -\frac{\hbar^2}{2m}\nabla^2 + U_c(\vec{r}) + A|\varphi|^2$$

WITH:

$$A = (N-1)4\pi\hbar^2 \frac{a}{m}$$

(a = SCATTERING LENGTH)

Collisions or interaction are responsible for all nice properties

Example: excitation of the coherent modes

FIRST: CALCULATION OF GROUND AND EXCITED STATES

$$H(\varphi) = -\frac{\hbar^2}{2m}\nabla^2 + U_c(\vec{r}) + A|\varphi|^2$$

HARMONIC TRAP

$$U_c(\vec{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

UNITS ON NATURAL VARABLES

$$\omega_0 \equiv (\omega_x \omega_y \omega_z)^{1/3}, \qquad l_0 \equiv \sqrt{\frac{\hbar}{m\omega_0}}.$$

INTERACTION PARAMETER

$$g \equiv \frac{mA}{\hbar^2 l_0} = 4\pi(N-1)\frac{a}{l_0}.$$

External pumping $V_p = V(\vec{r})\cos\omega t.$

$$\Delta\omega \equiv \omega - \omega_{p0} \qquad \left|\frac{\Delta\omega}{\omega_{p0}}\right| \ll 1$$

Solutions: $\varphi(\vec{r},t) = \sum_n c_n(t)\varphi_n(\vec{r})\exp\left(-\frac{i}{\hbar}E_n t\right)$

Population of levels $n_j = n_j(t) \equiv |c_j(t)|^2,$

$$\alpha \equiv \frac{1}{\hbar}A_{0p0} = \frac{A}{\hbar}\int |\varphi_0(\vec{r})|^2|\varphi_p(\vec{r})|^2 d\vec{r}$$ -> INTERACTION

TRANSITION AMPLITUDE→ $\beta \equiv \frac{1}{\hbar}V_{0p} = \frac{1}{\hbar}\int \varphi_0^*(\vec{r})V(\vec{r})\varphi_p(\vec{r})d\vec{r}.$

EQUATIONS FOR THE TWO MAIN STATES

$$\frac{dc_0}{dt} = -i\alpha n_p c_0 - \frac{i}{2}\beta e^{i\Delta\omega t} c_p,$$

$$\frac{dc_p}{dt} = -i\alpha n_0 c_p - \frac{i}{2}\beta^* e^{-i\Delta\omega t} c_0,$$

$$\frac{d^2 c_0}{dt^2} + i(\alpha - \Delta\omega)\frac{dc_0}{dt} + \left[\frac{|\beta|^2}{4} - \alpha n_p(\alpha n_0 - \Delta\omega)\right] c_0 = 0,$$

$$\frac{d^2 c_p}{dt^2} + i(\alpha + \Delta\omega)\frac{dc_p}{dt} + \left[\frac{|\beta|^2}{4} - \alpha n_0(\alpha n_p + \Delta\omega)\right] c_p = 0,$$

ANALYTICAL SOLUTION FOR THE TWO POPULATIONS

$$c_0 = \left[\cos\frac{\Omega t}{2} + i\,\frac{\alpha(n_0 - n_p) - \Delta\omega}{\Omega}\sin\frac{\Omega t}{2} \right]$$

$$\times \exp\left\{ -\frac{i}{2}(\alpha - \Delta\omega)t \right\},$$

$$c_p = -i\,\frac{\beta^*}{\Omega}\sin\frac{\Omega t}{2}\exp\left\{ -\frac{i}{2}(\alpha + \Delta\omega)t \right\},$$

RABI TYPE FREQUENCY

$$\Omega^2 = [\alpha(n_0 - n_p) - \Delta\omega]^2 + |\beta|^2$$

NORMALIZED VARIABLES

$$b \equiv \frac{\beta}{\alpha}, \quad \varepsilon \equiv \frac{\delta}{\alpha} \qquad \delta \equiv \Delta\omega$$

EQUATIONS FOR THE TWO MAIN STATES

$$\frac{dc_0}{dt} = -i\alpha n_p c_0 - \frac{i}{2}\beta e^{i\Delta\omega t} c_p ,$$

$$\frac{dc_p}{dt} = -i\alpha n_0 c_p - \frac{i}{2}\beta^* e^{-i\Delta\omega t} c_0 ,$$

$$\frac{d^2 c_0}{dt^2} + i(\alpha - \Delta\omega)\frac{dc_0}{dt} + \left[\frac{|\beta|^2}{4} - \alpha n_p(\alpha n_0 - \Delta\omega)\right] c_0 = 0,$$

$$\frac{d^2 c_p}{dt^2} + i(\alpha + \Delta\omega)\frac{dc_p}{dt} + \left[\frac{|\beta|^2}{4} - \alpha n_0(\alpha n_p + \Delta\omega)\right] c_p = 0,$$

ANALYTICAL SOLUTION FOR THE TWO POPULATIONS

$$c_0 = \left[\cos\frac{\Omega t}{2} + i\,\frac{\alpha(n_0 - n_p) - \Delta\omega}{\Omega}\sin\frac{\Omega t}{2}\right]$$

$$\times \exp\left\{-\frac{i}{2}(\alpha - \Delta\omega)t\right\},$$

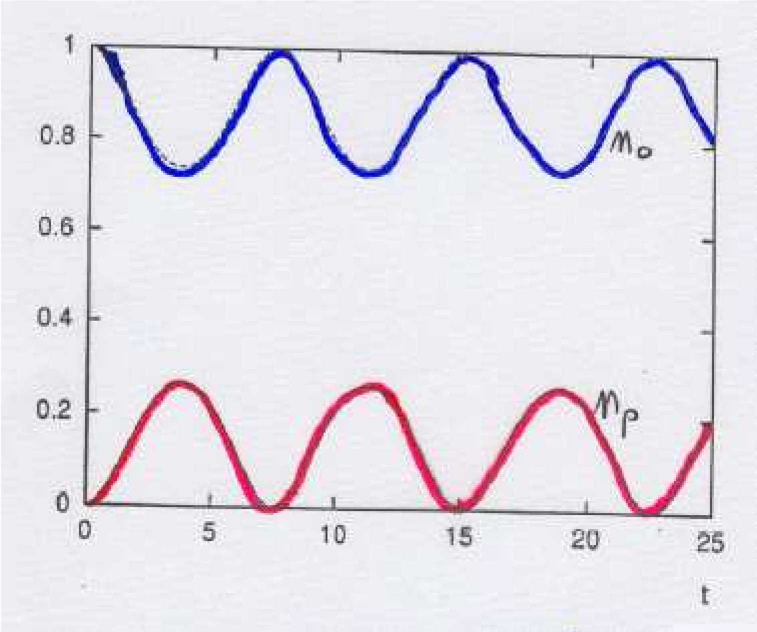
$$c_p = -i\,\frac{\beta^*}{\Omega}\sin\frac{\Omega t}{2}\exp\left\{-\frac{i}{2}(\alpha + \Delta\omega)t\right\},$$

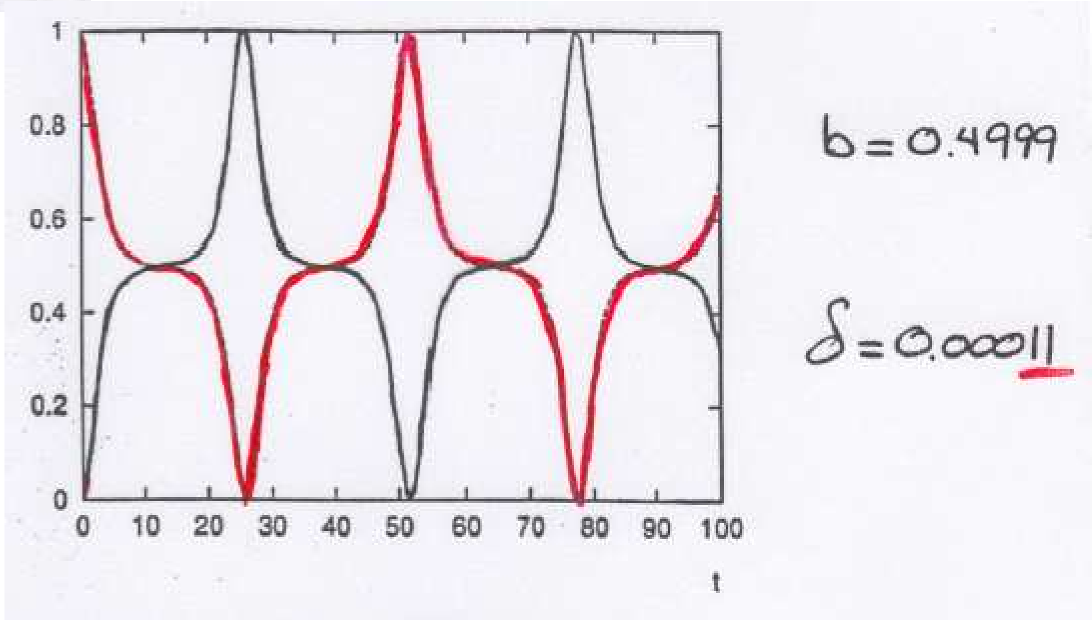
RABI TYPE FREQUENCY

$$\Omega^2 = [\alpha(n_0 - n_p) - \Delta\omega]^2 + |\beta|^2$$

NORMALIZED VARIABLES

$$b \equiv \frac{\beta}{\alpha}, \quad \varepsilon \equiv \frac{\delta}{\alpha} \qquad \delta \equiv \Delta\omega$$


1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
t
n0
np


1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
t
b = 0.4999
δ = 0.00011

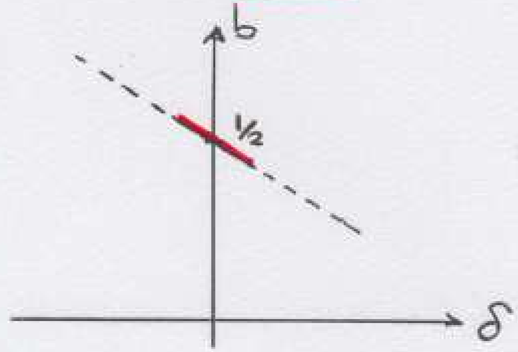
* This behaviour is typical of bifurcation

* Investigation to identify the bifurcation Line

$$b_c \cong 1/2 - \delta$$

$$b_c \cong \frac{1}{2} - \delta$$

Near $\delta \simeq 0$



$$b \equiv \frac{|\beta|}{\alpha} \qquad \delta \equiv \frac{\Delta\omega}{\alpha}$$

$\Longrightarrow$ **We have no equilibrium**

- Consider a time-averaged behavior

$\Downarrow$ Yukalov

Define an effective Hamiltonian

$$H_{eff} = \alpha n_0 m_p + \frac{1}{2}\left(\beta e^{i\Delta\omega t} c_0^* c_p + \beta^* e^{-i\Delta\omega t} c_0 c_p^*\right)$$

and the equations of motions

$$i\frac{dc_0}{dt} = \frac{\partial H_{eff}}{\partial c_0^*} \quad ; \quad i\frac{dc_p}{dt} = \frac{\partial H_{eff}}{\partial c_p^*}$$

→ taking time averagins

$$\overline{m}_p = \frac{b}{2\varepsilon}$$

where: $b = \frac{|\beta|}{\alpha}$ and $\varepsilon \equiv \frac{\overline{\Omega}}{\alpha}$ (dimentionless frequency)

→ in the effective Hamiltonian

$$E_{eff} = \frac{\alpha b^2}{2\varepsilon^2}\left(\frac{b^2}{2\varepsilon^2} + \delta\right)$$

$b \to 0$
$E_{eff} \to 0$

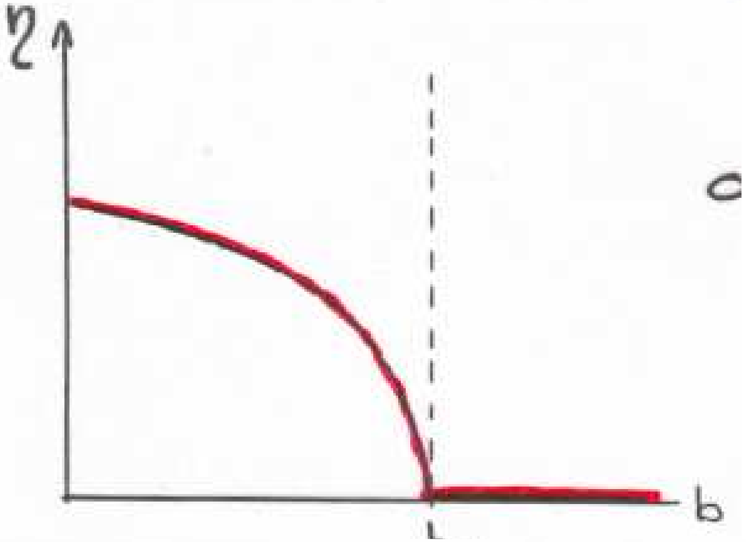
* order parameter → $\eta \equiv \overline{m}_0 - \overline{m}_p$

* heat capacity → $C = \frac{\partial E_{eff}}{\partial |\beta|}$

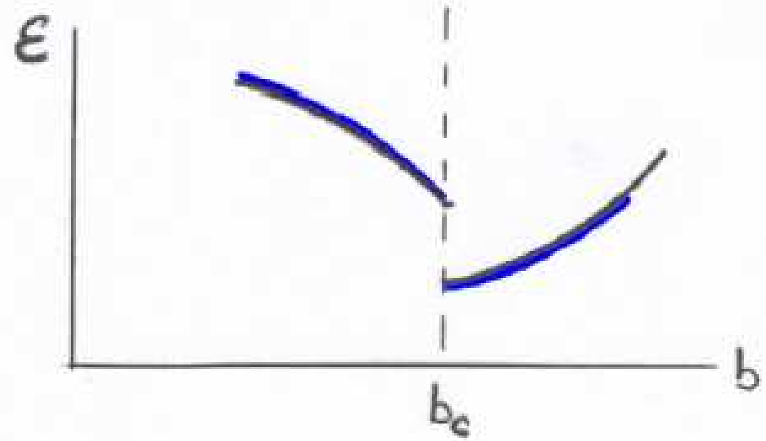
(capacity for the system to store pumped energy)

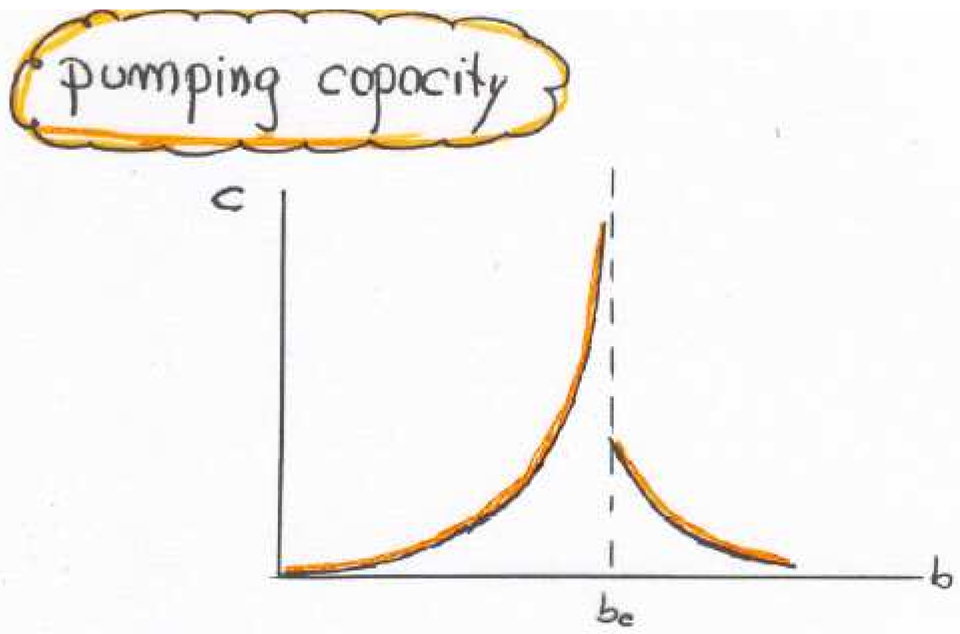
* Susceptibility → $\chi \equiv \left|\frac{\partial \eta}{\partial \delta}\right|$

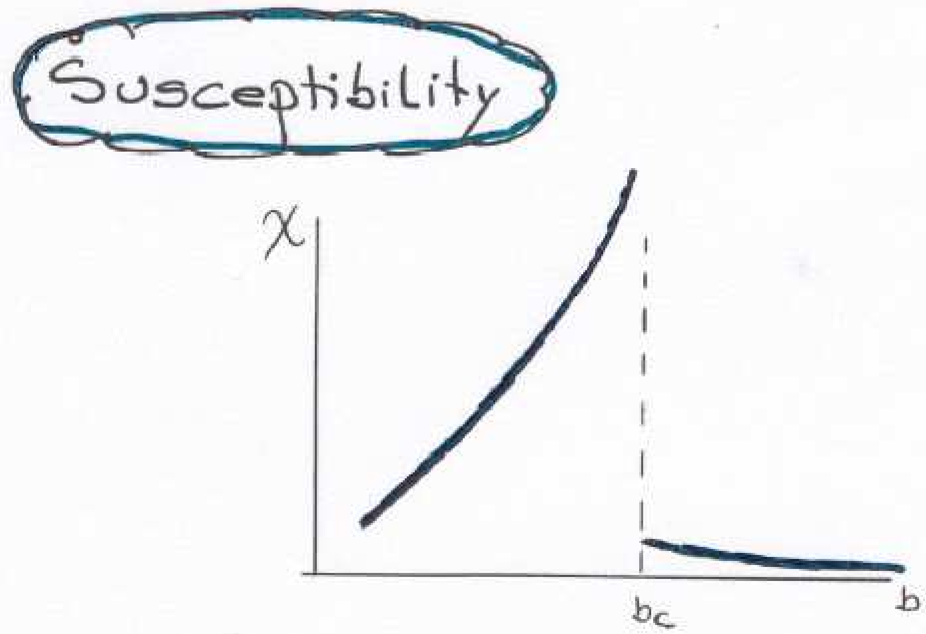
$\eta = \bar{M}_0 - \bar{M}_p$



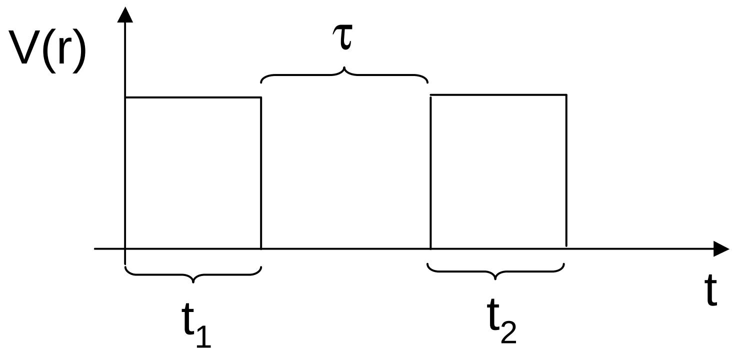
Averaged frequency

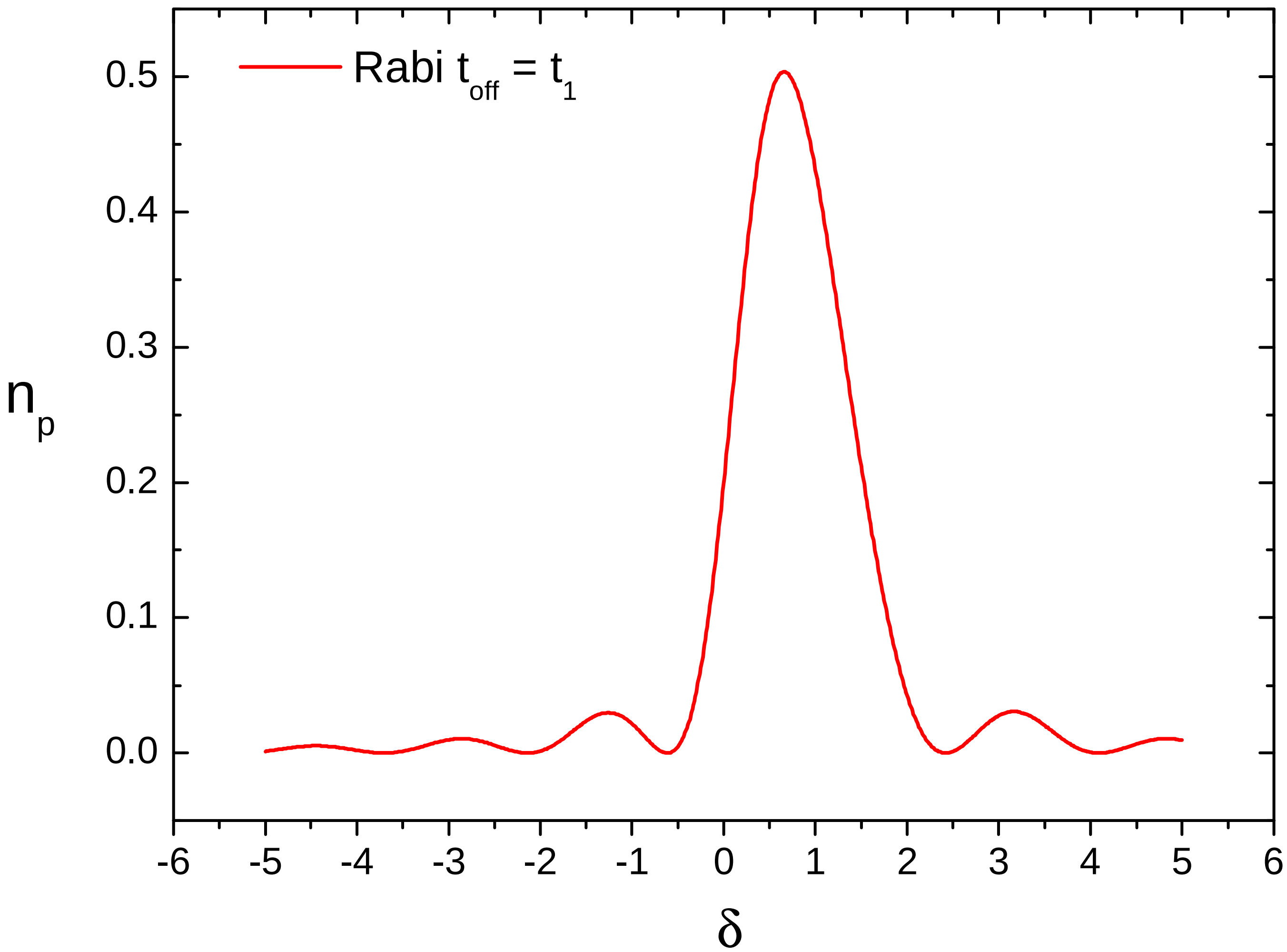



pumping capacity
c
b
b_c

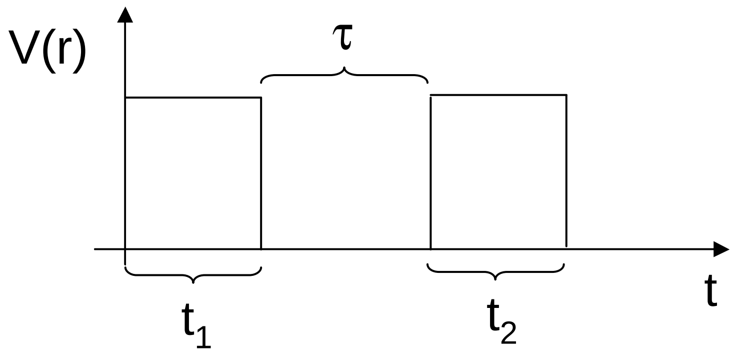

Susceptibility
χ
b
b_c

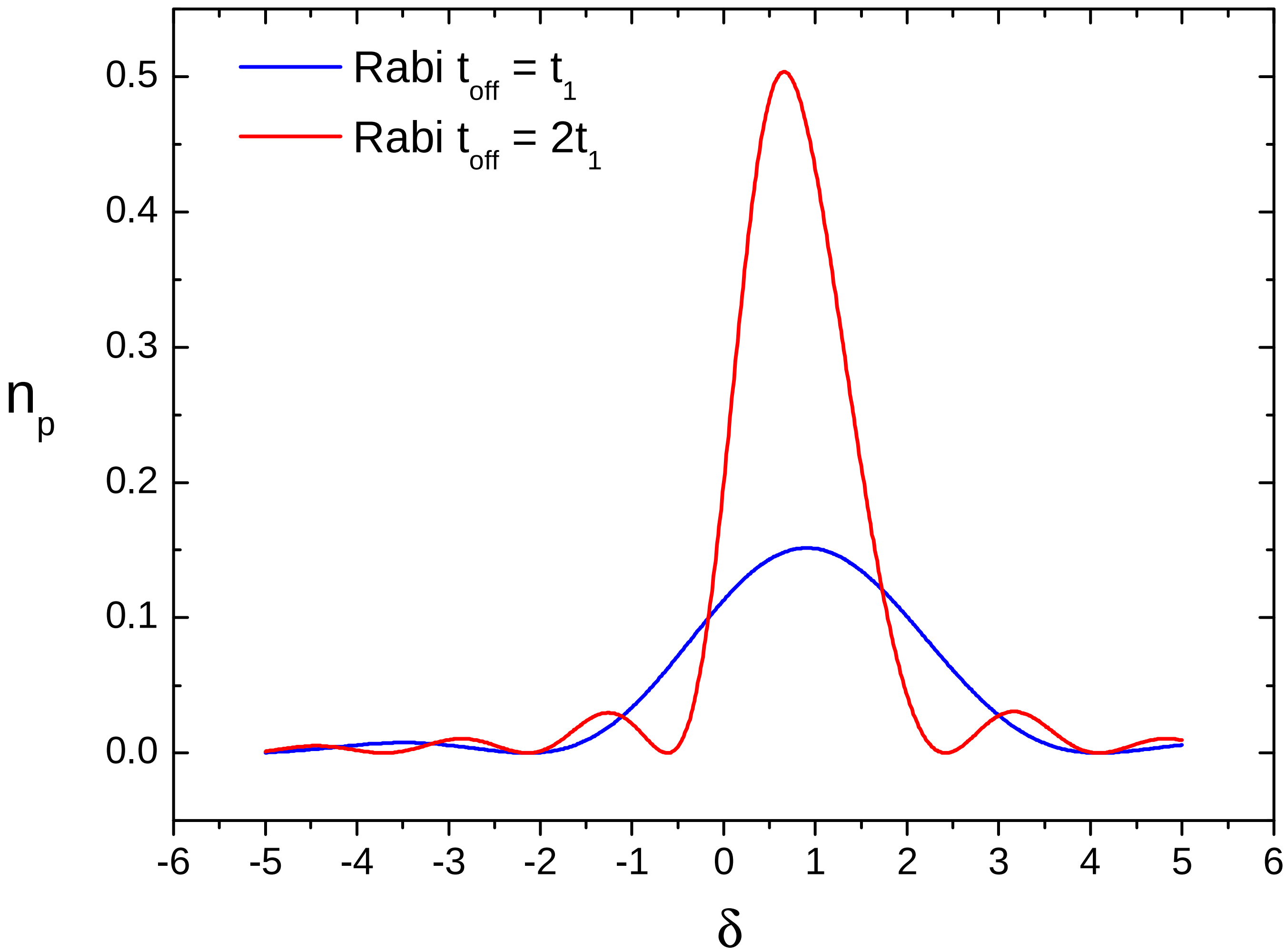
Ramsey Pulse
b=0.4



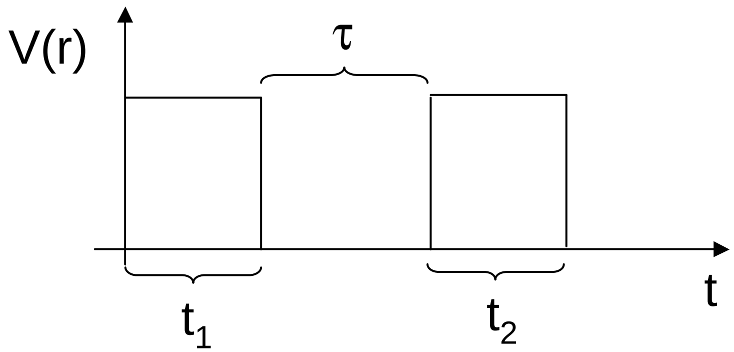


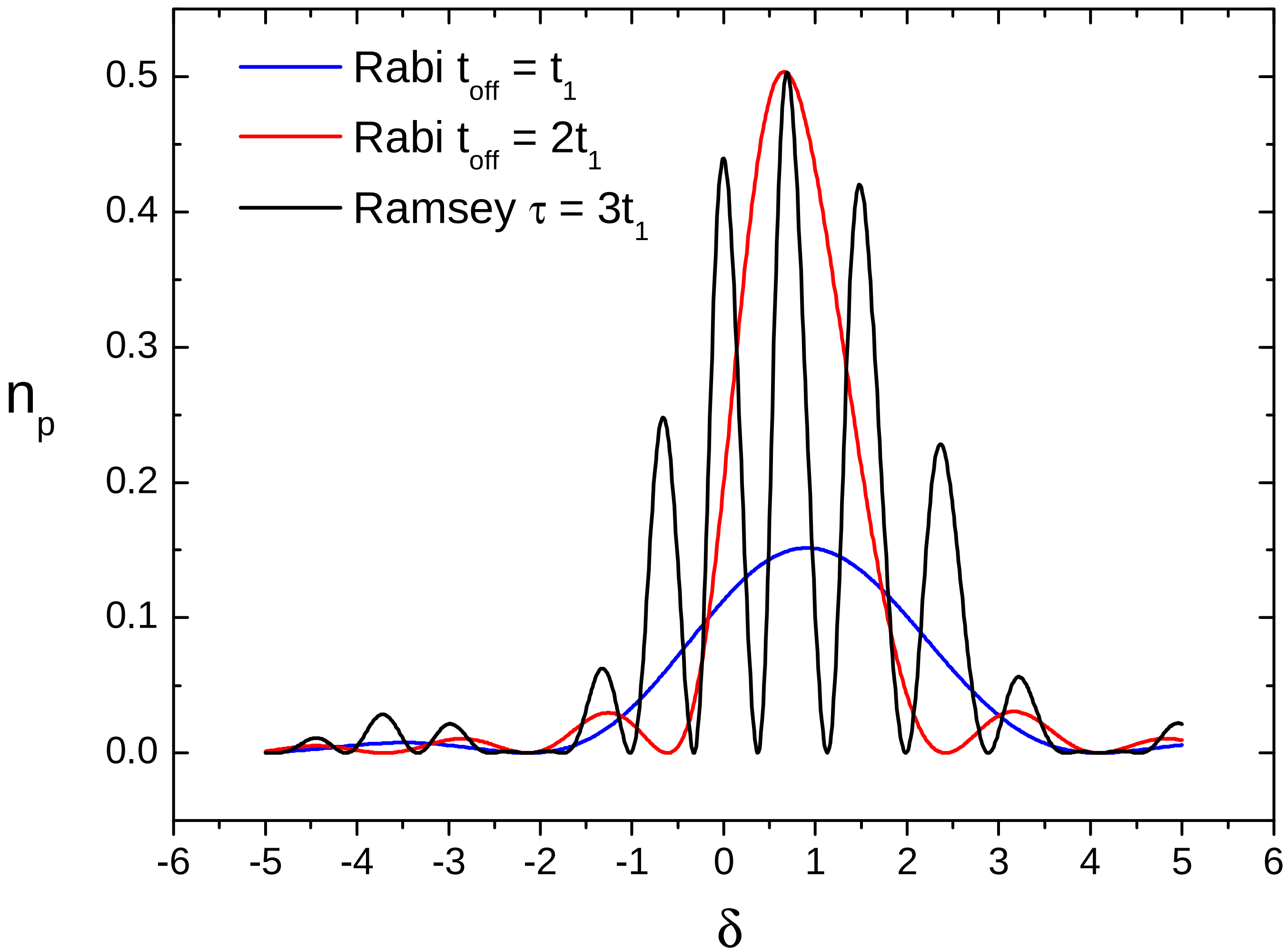
Ramsey Pulse
b=0.4



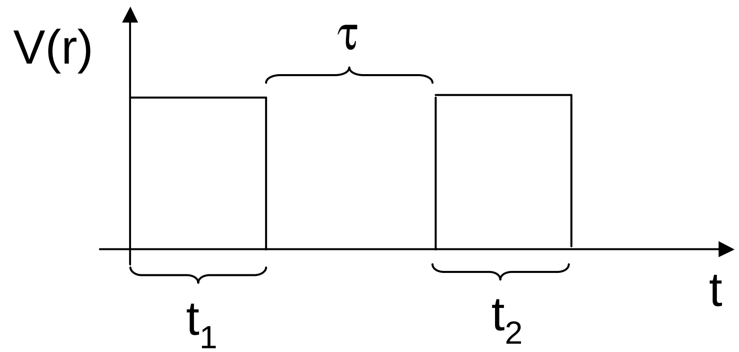


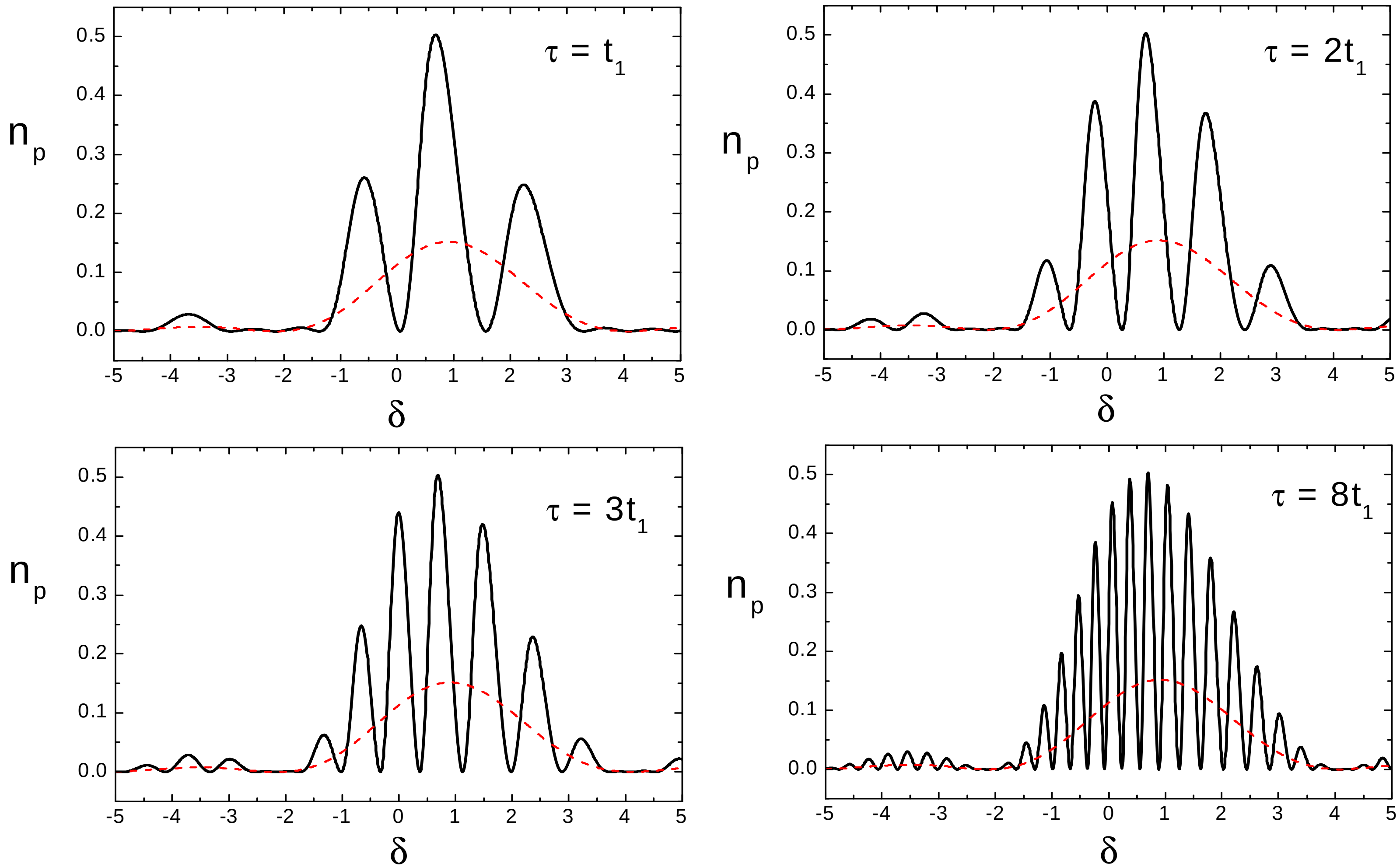
Ramsey Pulse
b=0.4

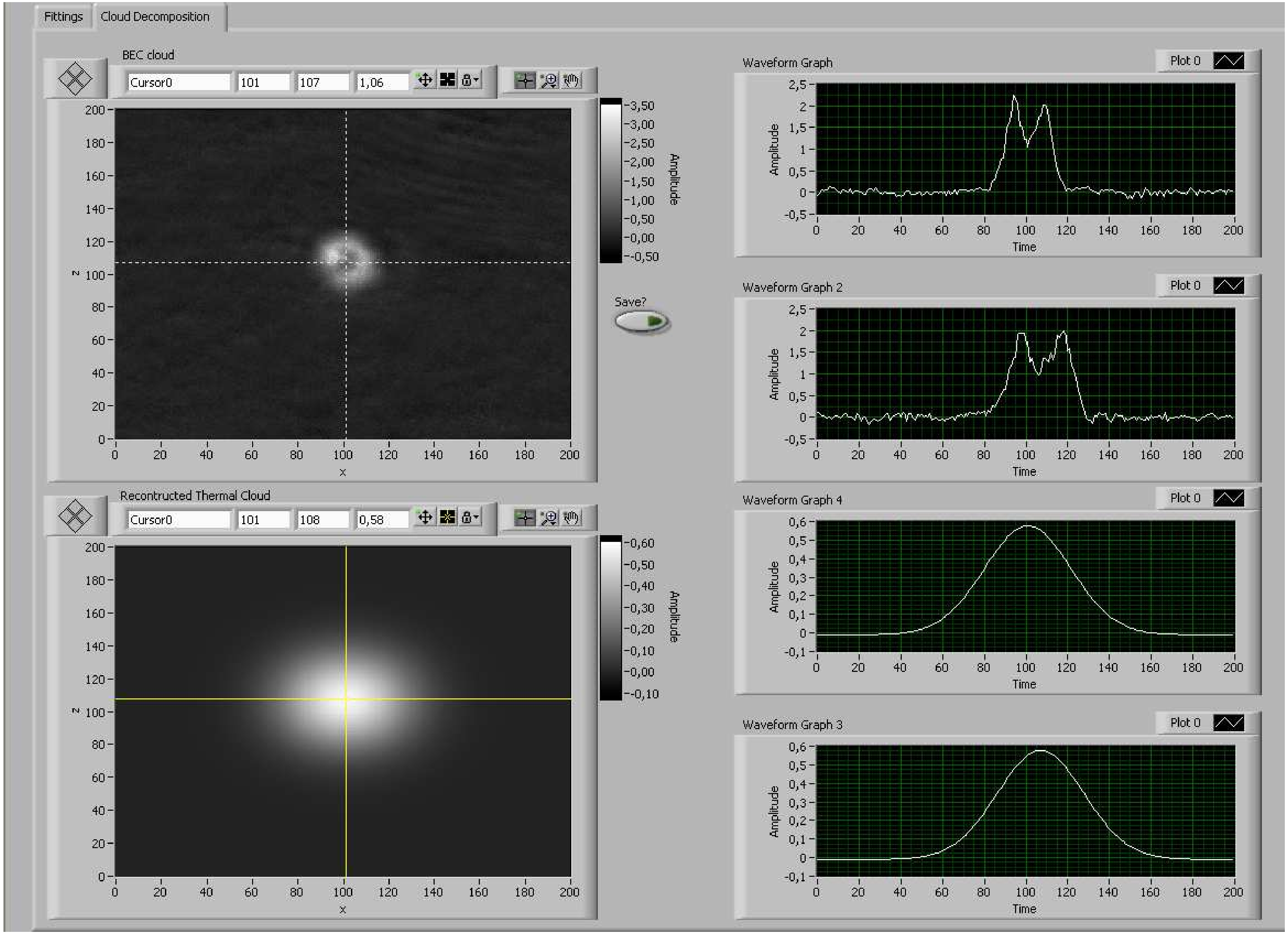


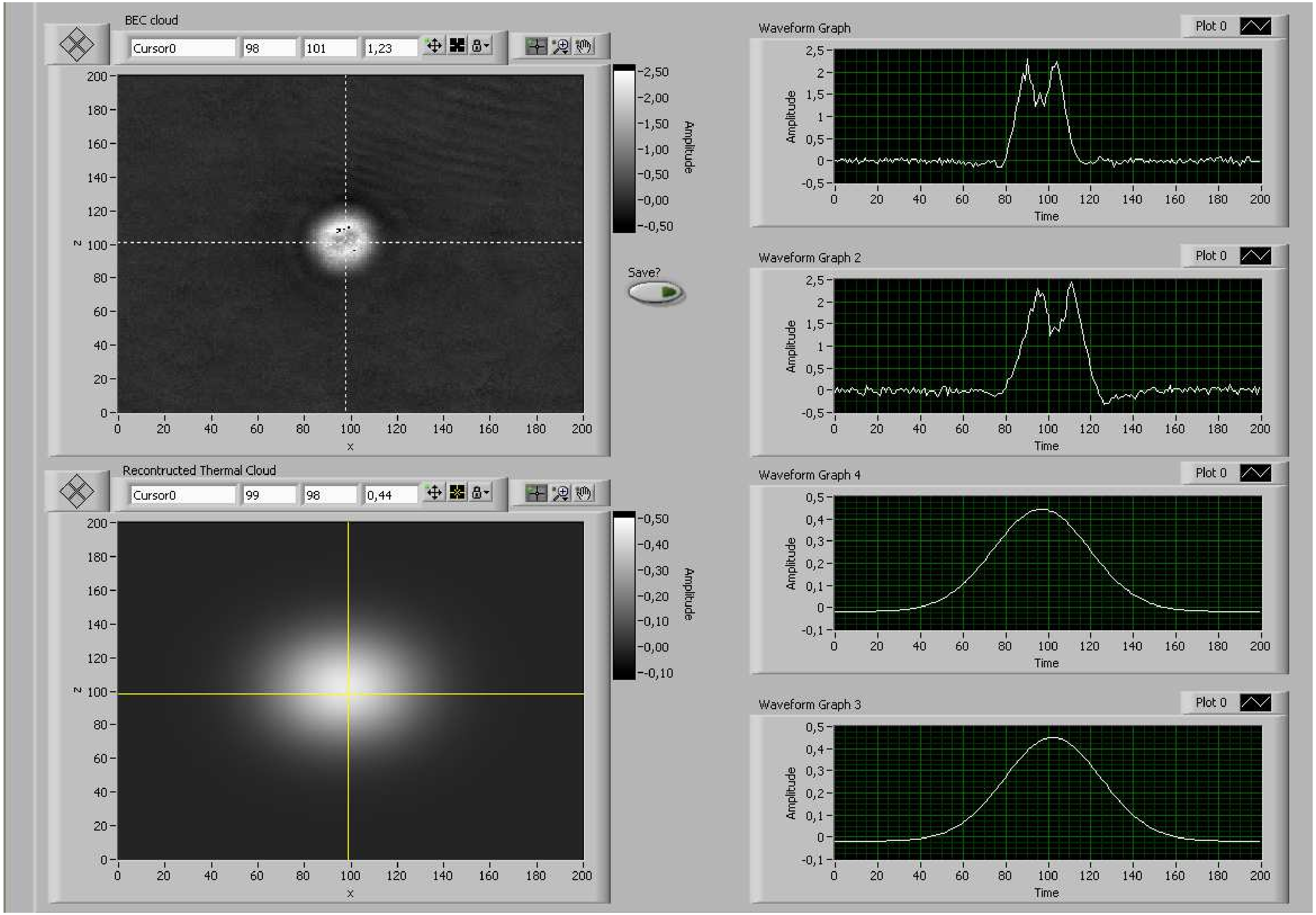


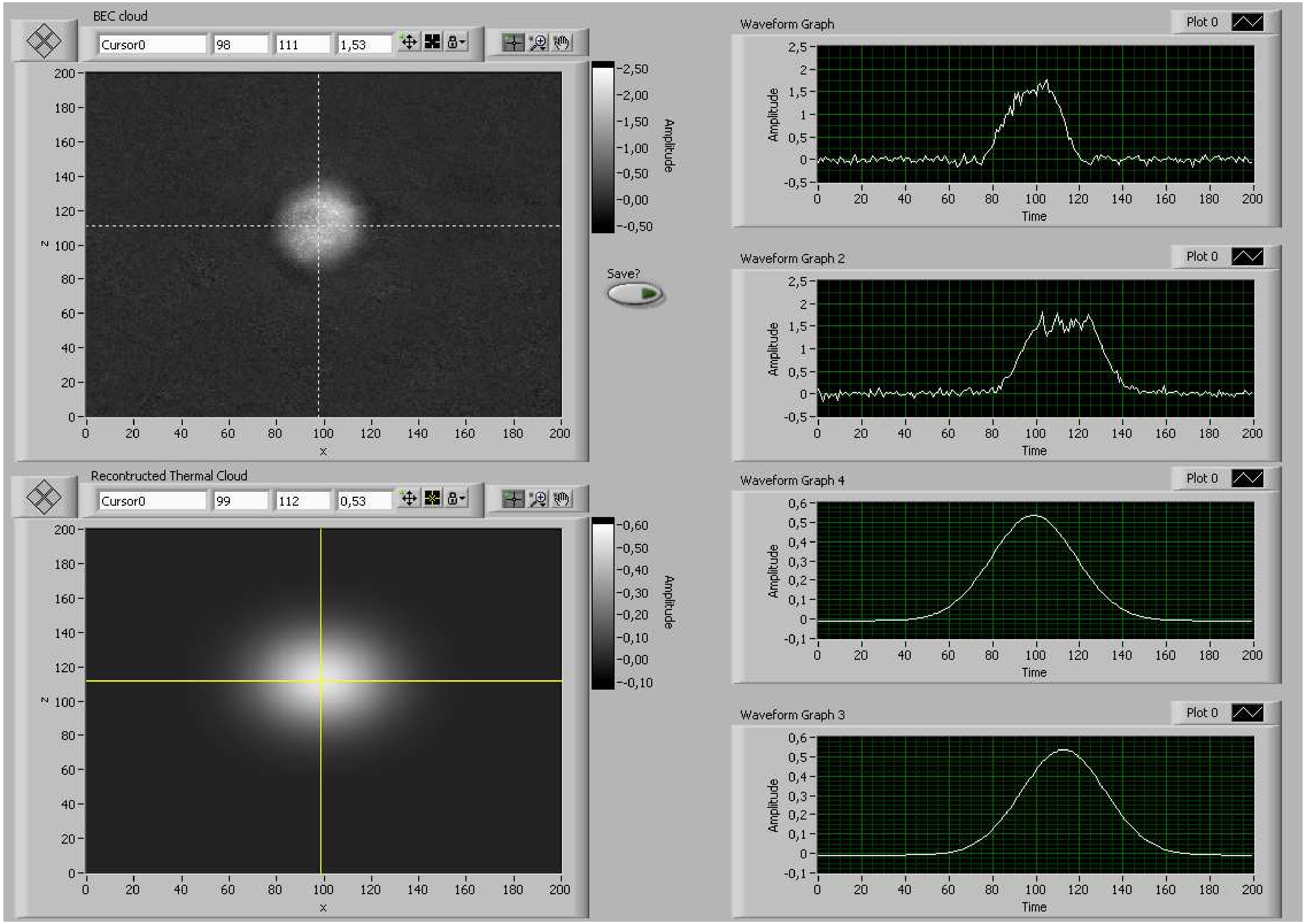
Ramsey Pulse
b=0.4





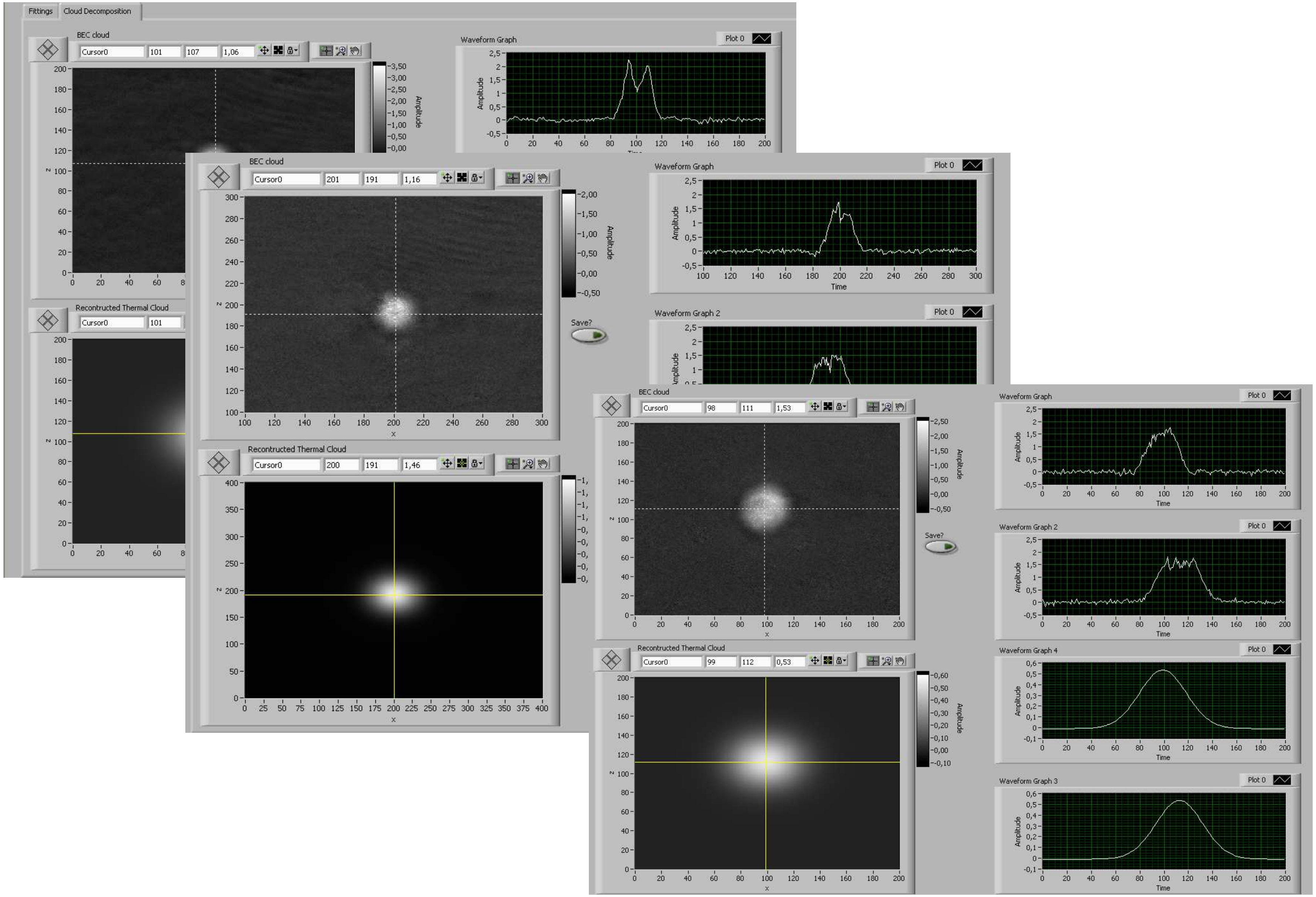

Fittings
Cloud Decomposition
BEC cloud
Cursor0
101
107
1,06
Amplitude
x
z
Save?
Recontructed Thermal Cloud
Cursor0
101
108
0,58
Amplitude
x
z
Waveform Graph
Plot 0
Amplitude
Time
Waveform Graph 2
Plot 0
Amplitude
Time
Waveform Graph 4
Plot 0
Amplitude
Time
Waveform Graph 3
Plot 0
Amplitude
Time


BEC cloud
Cursor0
98
101
1,23
Amplitude
Save?
Recontructed Thermal Cloud
Cursor0
99
98
0,44
Amplitude
Waveform Graph
Plot 0
Amplitude
Time
Waveform Graph 2
Plot 0
Amplitude
Time
Waveform Graph 4
Plot 0
Amplitude
Time
Waveform Graph 3
Plot 0
Amplitude
Time

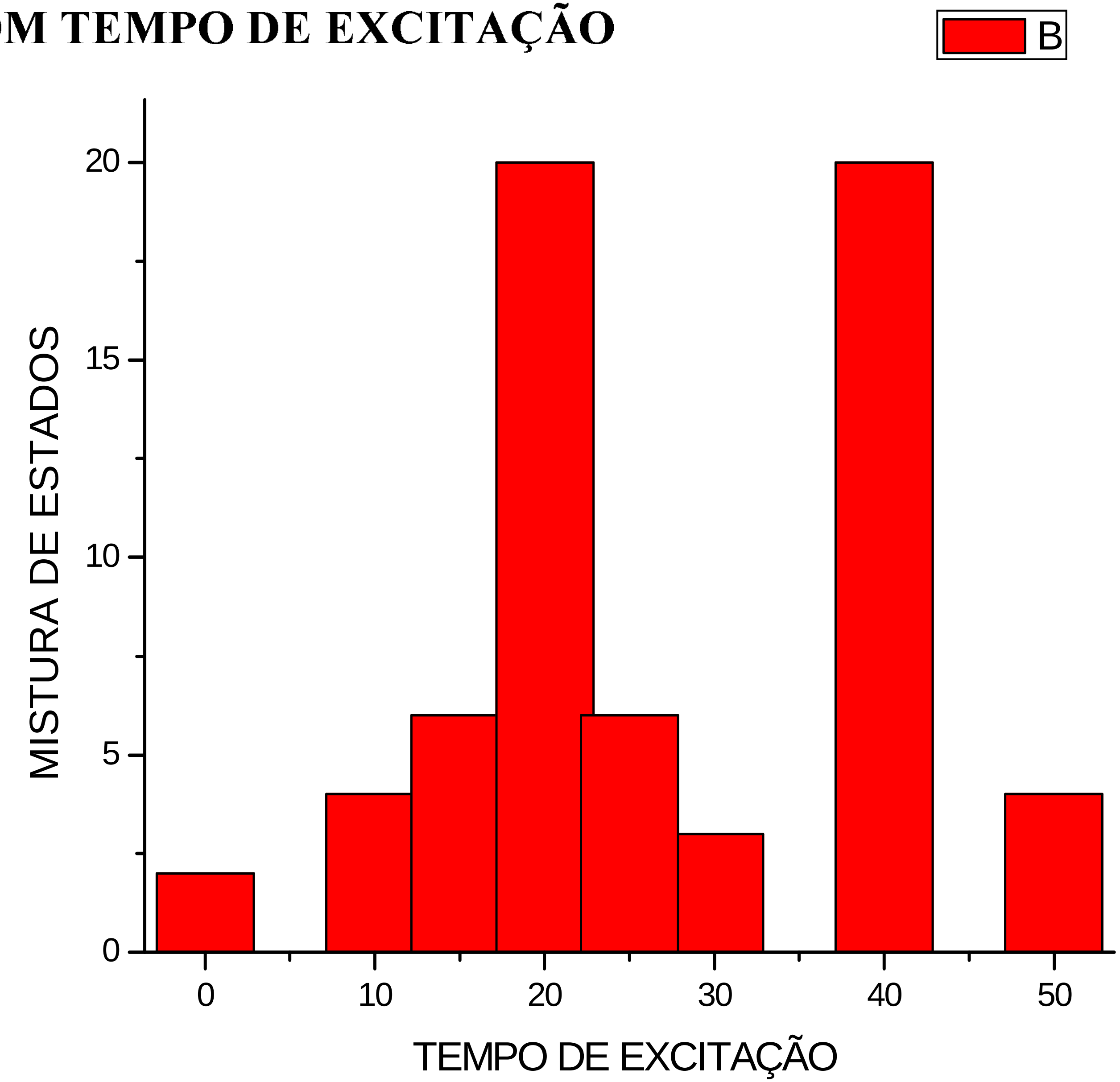


BEC cloud
Cursor0
98
111
1,53
Recontructed Thermal Cloud
Cursor0
99
112
0,53
Amplitude
Save?
Waveform Graph
Waveform Graph 2
Waveform Graph 4
Waveform Graph 3
Plot 0
Amplitude
Time
x
z

EVIDENCIAS DAS PRIMEIRAS OSCILAÇÕES TIPO RABI



PRESENÇA DE SUPERPOSIÇAO DE ESTADOS COM TEMPO DE EXCITAÇÃO



OSCILAÇÕES??????